The Role of Spatial Demand on Outlet Location and Pricing

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Abstract

In this paper we consider the problem of outlet pricing and location in the context of unobserved spatial demand. Our analysis constitutes a scenario wherein capacity-constrained firms set prices conditioned on their location, demand and costs. This enables firms to develop maps of latent demand patterns across the market in which they compete. The analysis further suggests locations for additional outlets and the resulting equilibrium effect on profits and prices.

Using Bayesian spatial statistics, we apply our model to seven years of data regarding apartment location and prices in Roanoke, Virginia. We find spatial covariation in demand to be material in outlet choice; the 95% spatial decay in demand extends 3.6 miles in a region measuring slightly over 9.5 miles. We further find that capacity constraints can cost complexes upwards of $100 per apartment. As predicted, price elasticities and costs are biased downward when spatial covariance in demand is ignored. Costs are biased upwards when ignoring capacity constraints. Using our analysis to suggest locations for entry, we find that a proper accounting of spatial effects and capacity constraints leads to an entry recommendation that improves profitability by 66% over a model that ignores these effects.

Keywords: Outlet Location, Pricing, Spatial Statistics, Structural Models, Competition.

1 Introduction

Outlet location and pricing are of central concern to many firms. For example, the French retailer Carrefour SA added 5930 stores between 1999 and 2003 while Dollar General in the United States added $2,426$ stores in the same period (Euromonitor 2005). Paramount to expansion efficacy is the effect of outlet location on sales, prices and profits, which are moderated by the underlying demand across regions in which a firm trades. Yet direct observation of demand across the areas in which firms trade is difficult: not only is this demand apportioned among existing outlets located at a small (relative to the trade space) number of fixed points in space, but the latent demand does not necessarily comport with the observed density of population. For example the presence of complementary stores or desirable traffic patterns may elevate demand in a specific locale.

Given the central role that the spatial distribution of demand plays in a firms' location decision, we focus on the inference of latent spatial demand. Integrating spatial statistics with a structural model of firm conduct allows us to capture a flexible distribution of latent spatial demand and use it to engage policy simulations regarding the sequential effect of locating an additional outlet on the demand, prices and profits for the new and existing outlets. This enables us to address questions such as the following:

- Can one infer the distribution of category demand across a given market or trade area from observations of retail sales at specific points in space? The answer to this question leads to a contour map of latent spatial demand that can be used to identify potential sites for new outlet entry or deletion.

- How do spatial demand and outlet location affect equilibrium prices and profits? We find that unobserved latent spatial demand leads to higher price variation over space, and that a proper accounting of spatial demand effects improves the profitability of an entry recommendation by 66% over a model that ignores spatial covariance in demand and costs.

There has been considerable prior work in marketing regarding outlet sales dating back to the gravity model (see, for example, Bucklin 1972; Ghose and Craig 1983). Our work differs from this research in a number of respects. First, this research typically assumes prices to be exogenous such that prices of competing outlets do not change with the location of an outlet. To the extent firms react to the new entry by changing prices, models that ignore this could misstate potential profits. In contrast, our model captures spatial endogeneity in pricing via a structural link between random spatial demand effects and equilibrium prices. Second, such models typically do not estimate latent demand across regions (i.e., the demand apportioned to an outlet arising from a particular point in space), but rather assume demand arises from observed differences in population across regions. In contrast, our work can accommodate unobserved sources of spatial demand. Third, we note that sales data necessary to estimate these models are often not observed as firms sometimes keep their outlet sales data private (e.g., Wal-Mart). Our approach need not require information on outlet sales to infer latent spatial demand.[1]

In this sense, our model of spatial demand is more related to analytical models of spatial location and demand in economics. Much of this work has been *theoretical*, assumes a uniform distribution for spatial demand, and focuses upon the equilibrium location of outlet location and the corresponding prices (Hotelling 1929; d'Aspremont et al. 1979; Ansari et al. 1994). Recently, *empirical* models in economics and marketing have begun to appear that focus on solving the *sub-game* of equilibrium prices or sales conditioned on outlet location and capacity in order to infer latent spatial demand (Chan et al. 2007; Pinske et al. 2002; Vankataraman and Kadiyali 2005; Thomadsen 2005, 2007; Davis 2001). This subgame is a reasonable starting point for the outlet location problem for a couple of reasons. First, it must be solved before determining the optimal outlet location. Second, in a preponderance of markets, outlet locations are extant and fixed at the time a late entrant decides to enter, so the relevant managerial decision pertains to locating the next outlet and its capacity. Using the sub-game, one can explore the implications of adding an outlet on equilibrium prices and demand for the new and existing outlets. Third, competitive response latencies in constructing outlets can be large due to land acquisition, zoning, permitting and construction. Therefore, competitive response in location may be impracticable over an intermediate duration suggesting a focus on the subgame is appropriate. A central innovation in these empirical models of spatial demand is that they consider observed spatial demand effects arising from distance to some centroid such as a population center or an airport. Our work complements the foregoing stream of research in a couple of key ways:

[1] By sales we refer to the observed number of units sold at a particular outlet. By demand, we refer to the distribution of product utility across various regions. It is the distribution of demand across space that drives sales at given locales.

- First, we supplement observed spatial demand factors via spatially correlated unobserved demand effects.[2] Given there are a plethora of potential spatial influences, it is unlikely that a researcher can capture all or most of them. For example, the presence of a particular employer, hotel, traffic pattern, school, restaurants, shops, family members, or friends could affect the choice of outlet and therefore equilibrium prices. Moreover, these influences tend to induce spatial covariance in demand shocks. The existence of spatial covariance in demand and supply shocks has a number of important implications for the outlet location problem:

  - Inclusion of unobserved spatial effects can enhance the efficiency of model estimates in information poor environments wherein covariates pertaining to spatial variation in demand are unobserved. We conjecture such cases are more common than those of complete information. As complete data are often prohibitively costly or unavailable our approach affords a feasible alternative in information poor environments.

  - The estimate of spatially correlated random effects in conjunction with spatial kriging yields a regional demand map which can serve as a decision aid to i) find new locations and ii) afford insights regarding differences in demand across the map (e.g., a peak in a certain area might suggest an important omitted variable).

  - We explicitly consider the case of apartments, where the observed population distribution can not be used to capture latent spatial demand as the population location itself is endogenous. Via the inclusion of latent spatial random effects we can capture such phenomenon. In other contexts such as retail outlets, our model of random spatial effects can be augmented using observed distances between consumers and the outlets or demographics.

  - For the reasons below, policy simulations pertaining to outlet location yield incorrect recommendations when spatial covariance is ignored. In our simulation, the policy recommendation from a model that incorporates spatial covariance yields a locale that would increase profits by 66% over a model that ignores spatial covariation.

[2] We differentiate between location-specific random effects (e.g., Bayer and Timmins 2007) commonly employed in sorting models and spatial random effects. Spatial models allow for spatial covariance in the location-specific effects. These covariances have implications for spatial prediction and outlet location and pricing. Our work further differs from this research stream inasmuch as we consider the supply side problem (price endogeneity) and capacity constraints in demand.

    * Spatial random effects are biased toward zero when spatial covariance is ignored, leading to a downward bias in estimates of price parameter. The bias arises as a result of the small sample properties inherent in Instrumental Variable estimation (Altonji and Segal 1996, Buse and Moazzami 1991). We demonstrate these effects via simulation and a data application. Given the number of outlets in a geographic trade area is often limited, the bias is consequential for the outlet location problem.
    * The downward bias in the price parameter also leads to a downward bias in the estimate for marginal costs. As price is the sum of costs plus markups, for a given price the lower estimated cost must be offset by a higher estimated markup.
    * The inclusion of spatial covariance in demand implies substitution patterns that are a function of entry location (apart from observed spatial differences). This is one approach to addressing the spatial IIA problem wherein expected share loss as a result of entry is apportioned not to proximal outlets but rather the largest ones.
    * In the absence of spatial covariance, predictions for demand are incumbent solely upon observed spatial differences and are likely to underestimate the true variation in demand. In our data the omission of spatial covariance attenuates the standard deviation of spatial demand by a factor of 11.

- Second, we consider the issue of outlet capacity, as supply is not limitless. Extant models do not accommodate the possibility that capacity might be constrained (as would be the case with categories such as restaurants, hotels, or apartments). This can be problematic for several reasons:

  - In the context of policy simulations, moving an outlet (or adding a new one) can not increase sales among extant outlets beyond their capacity.
  - An improper accounting of this constraint leads to biased estimates for marginal costs. When demand exceeds supply, firms can raise prices with no effect on sales. The high prices associated with at capacity firms leads to inferences of higher costs when capacity constraints are ignored. The additional error arising from poor predictions also leads to an increase in the estimated variance of the marginal cost equation in the supply-side model.

– Our approach yields estimates of the costs of these capacity constraints (via estimates of the Kuhn-Tucker multiplier), which can be used to assess the merits of expansion.

In sum, it is our goal to develop a flexible structural model of spatial demand in order to provide guidance to firms considering the potential location of new outlets (or changing the location of existing ones). In this sense, our work lies at the intersection of two developing research streams in marketing; empirical economics (Chintagunta et al. 2006) and spatial statistics (e.g., Bronnenberg and Mahajan 2001, Bronnenberg and Mela 2004, Bronnenberg and Sismeiro 2002). As such, we augment spatial statistics with structural models of pricing. Second we employ these spatial methods in a different context; outlet location. Third, given the emphasis on outlet location, we integrate capacity constraints into a spatial model.

The paper proceeds as follows. We begin by developing the demand-side model. We then generalize the model to consider the pricing problem faced by firms in the presence of capacity constraints, latent spatial demand, and the location of other firms. Next we outline how to estimate the model and, using these estimates, how to forecast demand prices and profits arising from the entry of a new outlet at any given location. We then describe the apartment data used to calibrate the model, and follow this with both simulated and data-based results. The simulated data are used to show the biases that arise when one omits spatial covariance and capacity constraints and the real data show these spatial effects lead to considerable improvements in model performance. We conclude the results of our data application, suggested locations for additional outlets and future research directions.

2 Model

Our model presentation proceeds as follows. First, we present consumer demand model. Specifically, we apply an individual-level, random utility, discrete choice model with spatially correlated random effects and use this model to infer the aggregate demand function. Second, we outline the equilibrium conditions of supply and demand in the context of firms who own outlets that are capacity constrained. The equilibrium conditions are derived from the profit maximizing behavior of the firms and utility maximizing behaviors of the consumers. The market clearing condition establishes the economic equilibrium of demand and supply which leads to the structural estimation equations. These equilibrium conditions constitute a system of equations that we estimate using advances in Bayesian spatial statistics.

2.1 Demand Model: Discrete Choice

Suppose there exist J outlets for a given type of goods (e.g., apartments, car dealers, hotels, bank branches, etc.) in a given region. The location of the $j \in J$ outlet is denoted as s_j. Let the number and the set of the potential customers for a set of outlets in a region be I, $i = \{1, ..., I\}$. Customer i's random indirect utility for choosing outlet j at location s_j in period t is

$$V_{tij} = X_{tj}\beta_1 + X_{ts_j}\beta_2 - P_{tj}\gamma_i + \theta_{ts_j} + \varepsilon_{tij}. \quad (1)$$

In this equation, X_{tj} represent the attributes of the j-th outlet such as variety or amenities; X_{ts_j} represents the *observed* attributes specific to the location (such as the distance to a business center or a major highway); and P_{tj} is a price index for an outlet. As is common in random utility models of choice, ε_{tij} is assumed independently drawn from a Gumbel distribution. Consumers' sensitivities to price change are assumed to be normally distributed across the population: we let $\gamma_i = \gamma + \eta_i$ where $\eta_i \sim N(0, \sigma_\gamma^2)$. We focus on price response heterogeneity to reduce the model's dimensionality. Gowrisankaran and Rysman (2007) find that adding random effects to non-price attributes i) leads to essentially no change in the model parameters and ii) that the random effects for these attributes do not differ from zero.

θ_{ts_j}, indexed by time t and location s_j, is the time varying random demand shock. We decompose $\theta_{ts_j} = \theta_{s_j} + \nu_{ts_j}$ where θ_{s_j} is a time-invariant spatial random effect that is fixed over all the observed periods and ν_{ts_j} is a spatial random effect that is independent across time.[3] This decomposition implies that there exists a systematic location-specific long-term effect (to capture unobserved spatial effects that do not vary over the range of the data, like the proximity to urban amenities, distance to major employers, school zoning and so forth) and a short-term perturbation about this mean, ν_{ts_j}, to capture unobserved spatial factors that may vary over time (e.g., construction and traffic patterns). As a result, both θ_{s_j} and ν_{ts_j} may be spatially correlated. We assume

[3] We considered other specifications for the temporal dependency in spatial shocks. In particular, we computed $\rho(v_{ts_j}, v_{t-1s_j})$ for each of the J firms for each of the draw of the v_{ts_j} in the MCMC sampling chain described in Appendix I. The mean ρ is 0.16 and the variance is 0.23. Hence, autocorrelations appear statistically small. We conjecture the large dispersion in autocorrelations results from having a limited number of periods in our data (six periods) from which to compute them.

$\theta \triangleq \left(\theta_{s_j}, j=1,\ldots,J\right) \sim N\left(0, \sigma_\theta^2 R_J\left(\phi_\theta\right)\right)$ and $\nu_t \triangleq \left(\nu_{ts_j}, j=1,\ldots,J\right) \sim N\left(0, \sigma_\nu^2 R_J\left(\phi_\nu\right)\right)$, whose joint effect is

$$\theta_t = \theta + \nu_t \sim N\left(0, \sigma_\theta^2 R_J\left(\phi_\theta\right) + \sigma_\nu^2 R_J\left(\phi_\nu\right)\right) \tag{2}$$

The entries of the correlation matrix $R_J\left(\phi\right)$ ($\phi = \phi_\theta$ or ϕ_ν) can be constructed with an isotropic exponential decay function, $\exp\left(-\phi \cdot d\right)$ where d is the distance between any two locations: $d = ||s_i - s_j||$; the Matern class (Banerjee et al. 2004); or a more general anisotropic nonparametric spatial Dirichlet process models (Gelfand et al. 2005; Duan et al. 2007). We note that these spatial structures are highly flexible and admit many potential latent demand surfaces.

The customer i may choose the outside good, that is she may not buy in the region at all or she may consider expenditures on other types of goods sold in the region. Her indirect utility for the outside good is given by

$$V_{ti0} = M_t + \varepsilon_{ti0}. \tag{3}$$

If the highest utility of choosing one outlet exceeds that of the outside-good, the customer will select the highest utility outlet.

The customer chooses the outlet with the highest utility. Defining the mean effects in the utility function as $\xi_{tj} = X_{tj}\beta_1 + X_{ts_j}\beta_2 - P_{tj}\gamma + \theta_{ts_j}$, then the customer utility function in equation (1) can be rewritten as $V_{tij} = \xi_{tj} - P_{tj}\eta_i + \varepsilon_{tij}$, where η_i and ε_{tij} capture consumer heterogeneity. We assume that the distributions for the random effects ε_{tij} and η_i are common knowledge to firms. This assumption is necessary for firms to form expectations regarding their market share. The Gumbel error assumption for ε_{tij} results in a logit choice likelihood that customer i chooses outlet j:

$$W_{tij} = \frac{e^{\xi_{tj} - P_{tj}\eta_{3i}}}{\sum_{l=1}^{J} e^{\xi_{tl} - P_{tl}\eta_{3i}} + e^{M_t}}.$$

The expected market share for outlet j is W_{tj} is obtained by integrating over the remaining random effect, η_i, yielding

$$W_{tj} = \int \frac{e^{\xi_{tj} - P_{tj}\eta_i}}{\sum_{l=1}^{J} e^{\xi_{tl} - P_{tl}\eta_i} + e^{M_t}} dF\left(\eta_i\right). \tag{4}$$

Multiplying the number of persons in the market, I, by the j's share leads to the total expected demand, $Q_{tj} = I \times W_{tj}$. The second term in the denominator reflects the demand for the outside good. As I, M_t and the intercept of the additive utility cannot be separately identified at the same time, we make I and M_t constant as discussed in the Data section.

Berry, Levinshon and Pakes (1995, henceforth BLP) prove that W_{tj} and ξ_{tj} is a one-to-one mapping conditioning on the distribution of η_i by a contraction mapping theorem. The natural algorithm derived from the contracting mapping to compute ξ_{tj} by setting the observed $\hat{W}_{tj} = W_{tj}$ is given as

$$\xi_t^{(g+1)} = \xi_t^{(g)} + \ln \hat{W}_t - \ln W_t \left(\xi_t^{(g)}, F(\eta_i) \right). \tag{5}$$

This algorithm inverts the vector function (5) from the observed shares $\hat{W}_t$ to the unobserved errors $\hat{\xi}_t$ (asymptotically approached by $\xi_t^{(g)}$). This inversion leads to a nonlinear transformation of the data given that $F(\eta_i) = N\left(0, \sigma_\gamma^2\right)$. Note that $\hat{\xi}_t$ is linear in the demand-side model parameters (β, γ, and θ) which facilitates the demand-side estimation.

2.2 Supply Model: Bertrand-Nash Game

In addition to consumers, the market comprises firms who compete on the basis of price, location and capacity. As noted previously, we focus upon the subgame of firm competition conditioned on location choice and capacity. Assume a market is comprised of F firms wherein each firm f has a set F_f of outlets. Each firm maximizes the total profit of its outlets F_f. Let us first consider firm f's strategy. Conditioning on the prices of the outlets not belonging to f's chain, firm f faces the following profit maximization problem:

$$\max \Pi_{tf} = \sum_{j \in F_f} (P_{tj} - c_{tj}) Q_{tj} \text{ s.t. } Q_{tj} \leq K_j \tag{6}$$

where c_{tj} represents the variable cost of goods sold and K_j is outlet j's capacity constraint of K_j, which is the total number of goods that the outlet can produce, sell or rent. The capacity is considered rigid over the short decision frame of a firm setting prices as the expansion or contraction of an outlet's physical plant involves construction/permitting time and considerable cost. This implies that the capacity consideration is relegated to an "upper stage" of the game wherein a firm sets location and capacity prior to choosing price in the subgame. When facing a different demand at time t as a result of setting the price P_{tj}, K_j becomes an immutable short-term constraint to which the outlet is subject.

As the firms' variable costs c_{tj} are not observed by the econometrician, we model them by

$$c_{tj} = Y_{tj} \beta_3 + \zeta_{ts_j} \tag{7}$$

where Y_{tj} are a set of cost shifters (for example, the type of inventory). ζ_{ts_j} are spatially correlated cost shocks. We further decompose the time varying cost shock as $\zeta_{ts_j} = \zeta_{s_j} + e^c_{ts_j}$ where ζ_{s_j} is a time-invariant random effect that is fixed over the observed periods and $e^c_{ts_j}$ is the cost shock that is independent across time. ζ_{s_j} captures the long-term cost factors which are not observed and controlled in cost shifters Y_{tj}. ζ_{s_j} can include the labor, maintenance and administrative costs, tax, insurance and other unobserved factors, whereas $e^c_{ts_j}$ is the short-term changes in these factors. Accordingly ζ_{s_j} and $e^c_{ts_j}$ are likely to be spatially correlated as a result of these omitted variables in the cost model. We assume $\zeta \triangleq \left(\zeta_{s_j}; j = 1, \ldots, J\right) \sim N\left(0, \sigma^2_\zeta R_J\left(\psi_\zeta\right)\right)$ and $e^c_t \triangleq (e^c_{ts_j}; j = 1, \ldots, J) \sim N\left(0, \sigma^2_c R_J\left(\psi_c\right)\right)$. $R_J\left(\psi\right)$ ($\psi = \psi_\zeta$ or ψ_c) can be of the same or different functional form of the correlation in spatial demand $R_J\left(\phi\right)$.

The Kuhn-Tucker conditions for this optimization problem with inequalities constraints are

$$\frac{\partial \Pi_{tf}}{\partial P_{tk}} = \sum_{j \in F_f} \left(P_{tj} - Y_{tj}\beta_3 - \zeta_{tj}\right) \frac{\partial Q_{tj}}{\partial P_{tk}} + Q_{tk} - \sum_{m \in F_f} \lambda_{tm} \frac{\partial Q_{tm}}{\partial P_{tk}} = 0; k \in F_f \tag{8}$$

with the Kuhn-Tucker multipliers, $\lambda_{tm} \geq 0$, and

$$\begin{array}{c} Q_{tm} = K_m, \text{ iff } \lambda_{tm} > 0 \\ \text{and } Q_{tm} < K_m, \text{ iff } \lambda_{tm} = 0 \end{array}. \tag{9}$$

These multipliers reflect the marginal cost of the capacity constraint on apartment profitability and reflect the value of expanding a particular outlet's capacity by one unit.

As $Q_{tj} = I \times W_{tj}$, the representation of (8) with market shares is

$$\sum_{j \in F_f} \left(P_{tj} - Y_{tj}\beta_3 - \zeta_{tj}\right) \frac{\partial W_{tj}}{\partial P_{tk}} + W_{tk} - \sum_{m \in F_f} \lambda_{tm} \frac{\partial W_{tm}}{\partial P_{tk}} = 0; k \in F_f. \tag{10}$$

The Kuhn-Tucker conditions imply that firm f chooses optimal prices P^*_{tk} for the outlets with expected vacancy and the second best price $\tilde{P}_{tm}$ for outlets at capacity. P^*_{tk} is the solution of the first order condition (10) above with $\lambda_{tk} = 0$, whereas $\tilde{P}_{tm}$ satisfies the binding capacity constraint (9). P^*_{tk}, $\tilde{P}_{tm}$ and the non-zero multiplier λ_m are solved simultaneously.

There are $|F_f|$ ($|F_f|$ denotes the number of outlets in F_f) equations arising from the first order conditions for each firm. To facilitate explication, we define a $|F_f| \times |F_f|$ matrix whose (j,k)-th element is

$$\Omega^{(t,f)}_{jk} = \frac{\partial W_{tj}}{\partial P_{tk}} = \left\{ \begin{array}{c} -\int \left(\gamma + \eta_i\right) W_{tij} \left(1 - W_{tij}\right) dF\left(\eta_i\right), \text{ for } j = k \in F_f \\ \int \left(\gamma + \eta_i\right) W_{tij} W_{tik} dF\left(\eta_i\right), \text{ for } j \neq k \in F_f. \end{array} \right. . \tag{11}$$

Let the vectors $\lambda_f = (\lambda_m, m \in F_f)$, $P_{tf} = (P_{tj}; j \in F_f)$, $W_{tf} = (W_{tj}; j \in F_f)$, $\zeta_{tf} = \left(\zeta_{ts_j}, j \in F_f\right)$ and the submatrix $Y_{tf} = (Y_{tj}, j \in F_f)$. Note for the outlets with spare capacity $\lambda_m = 0$ and for the outlets with full capacity $W_{tj} = \frac{K_j}{I}$. With these definitions, the first order conditions (10) can be rewritten in matrix form as

$$\Omega^{(t,f)} \left(P_{tf} - Y_{tf}\beta_3 - \lambda_{tf}\right) + W_{tf} = \Omega^{(t,f)}\zeta_{tf}.$$

Likewise, for all the J outlets belonging to F firms in the market, we define the following $J \times J$ matrix whose (j,k)th element is,

$$\Omega_{jk}^{(t)} = \begin{cases} \frac{\partial W_{tj}(P_{tj}, P_{t,-j}, X_t, \theta_t)}{\partial P_{tk}}, \text{ for } j,k \in \text{ same } F_f \\ 0, \text{ otherwise} \end{cases}. \tag{12}$$

The first order conditions for all the outlets can be written in matrix form as

$$\Omega^{(t)} \left(P_t - Y_t\beta_3 - \lambda_t\right) + W_t = \Omega^{(t)}\zeta_t. \tag{13}$$

If $\Omega^{(t)}$ is invertible, then we have

$$P_t - \lambda_t + {\Omega^{(t)}}^{-1} W_t = Y_t\beta_3 + \zeta_t. \tag{14}$$

In equation (14) $B_t\left(\lambda_t, \gamma\right) \equiv \lambda_t - {\Omega^{(t)}}^{-1} W_t$ indicates the firms' markups as this expression represents the difference between the firm's prices and its costs. Note that $\lambda_{tm} = 0$ for the outlets that have vacancy and $\lambda_{tm} > 0$ for the outlets with full occupancy. Hence, markups increase when firms are capacity constrained. The economic interpretation of this equation is that the prices of the outlets with full capacity are higher than their optimal prices. Intuitively, this suggests firms who face demand in excess of supply will raise prices to the point wherein demand is equal to capacity; lower prices only serve to decrease revenue.

Of interest, equation (14) embeds all parameters in the supply and demand system. This suggests it is possible to estimate both the demand-side and cost side parameters even in the *absence* of any demand-side data (Thomadsen 2005). Like BLP and others, we assume that these parameters, though unobserved to the econometrician, are known to the firm and to the consumers.

3 Estimation

We follow a two-stage estimation approach analogous to two stage least squares as in BLP and Nevo (2001). In the first stage we estimate the demand-side model to obtain estimates for γ, $\Omega^{(t)}$, and W_t which we denote $\widehat{\gamma}$, $\widehat{\Omega}^{(t)}$, and $\widehat{W_t}$. We then treat these estimated demand-side parameters as random variables in the supply-side model to obtain the estimates for costs and the Lagrange multipliers.

In spite of the loss of statistical efficiency that would be gained by using one-stage approach to estimate the demand side parameters (i.e, using information from both the demand and supply model to infer the demand parameters), the two-stage approach offers a number of advantages over a one-stage approach. First, a two stage approach mitigates the need to use instrumental variable (IV) estimation on the supply-side.[4] Second, the Jacobian required for joint estimation is quite complex rendering estimation substantially more cumbersome. Third, should the supply side model be mis-specified the resulting demand side parameters will be biased (Chintagunta, Kadiyali and Vilcassim 2006).

3.1 Demand-side Estimation

The demand-side estimation equations for all the outlets in the market are

$$W_{tj} = \frac{Q_{tj}}{I} = \int \frac{e^{\xi_{tj} - P_{tj}\eta_i}}{\sum_{l=1}^{J} e^{\xi_{tl} - P_{tl}\eta_i} + e^{M_t}} dF\left(\eta_i\right); \tag{15}$$

where $Q_{tj} = K_j$ if the outlet is at capacity (e.g. a full outlet) in that period. There is no closed form for the integral in equation (15). However, given the distribution of random price effects, $F\left(\eta_i\right)$, the integration involved to compute $W_t\left(\xi_t^{(g)}, F\left(\eta_i\right)\right)$ can be approximated via a Monte Carlo integration. The integration proceeds by simulating I_S "individuals" η_i from the distribution $F\left(\eta_i\right)$ and approximating the integral in (15) by

$$W_{tj}\left(\xi_{tj}\right) = \sum_{i=1}^{I_s} \frac{e^{\xi_{tj} - P_{tj}\eta_i}}{\sum_{l=1}^{J} e^{\xi_{tl} - P_{tl}\eta_i} + e^{M_t}}. \tag{16}$$

Instead of using a maximum likelihood method to estimate model parameters, BLP propose an instrumental variables-based approach. An appealing aspect of this approach is that unobserved effects impact both spatial demand and prices and the IV approach allows one to control for this

[4] The need for IV estimation in joint inference for γ arises because prices in the supply-side model of equation (14) are a function of $\Omega^{(t)}(\gamma, P_t(\zeta_t))$ and $W_t(\gamma, P_t(\zeta_t))$ which depend on prices which in turn depend on the cost shocks. Hence $\Omega^{(t)}$ and W_t are endogenous to the errors in the cost equation. This suggest the need for IV estimation when they are unknown. Of note, Ω and W enter the pricing model in a non-linear fashion. The properties of non-linear IV estimators are not clear; as a result we can not be sure an IV approach leads to unbiased estimates for γ.

possibility by forming instruments for price. Stacking the ξ_{tj} in equation (16) across j, let

$$\xi_t = X_t\beta_1 + X_{ts}\beta_2 - P_t\gamma + \theta + \nu_t. \tag{17}$$

The random spatial errors θ and ν_t in (17) can be structurally correlated with the prices P_{tj} as a result of the endogeneity of price to random spatial demand effects; prices tend to increase when demand is high.

Our goal is therefore to compute ξ_t from equation (16) so we can use the orthogonality conditions between the spatial errors ν_t in (17) and instruments Z_t to estimate β_1 and β_2 (that is, $E(Z_t^T\nu_t) = 0$). BLP prove that estimates $\hat{\xi}_t$ are computed using the iterative procedure, $\hat{\xi}_t^{(m+1)} = \hat{\xi}_t^{(m)} + \ln \hat{W}_t - \ln W_t\left(\hat{\xi}_t^{(m)}\right)$ by showing this equation is a contraction mapping. Once we obtain $\hat{\xi}_t$ (approximated by $\xi_t^{(m)}$ as the sequential discrepancy between $\xi_t^{(m)}$ and $\xi_t^{(m+1)}$ approaches zero), we consider the linear model

$$\nu_t = \hat{\xi}_t(\gamma) - X_t\beta_1 - X_{ts}\beta_2 + P_t\gamma - \theta \tag{18}$$

which is mean independent of the instruments Z_t and has a spatial covariance structure $N\left(0, \sigma_\nu^2 R_J(\phi_\nu)\right)$ given the true β_1, β_2, γ and θ.[5] We assume an exponential correlation function (Banerjee et. al. 2004) for the spatial random effects, that is $R_J(\phi) = \exp(-\phi \cdot ||s - s'||)$. This function implies an exponential decay in the covariance of spatial effects over distance.[6]

Using a likelihood approach (Chernozhukov and Hong 2003, Kim 2002, Romeo 2007), we let $Z_t^T\nu_t \sim N\left(0, \sigma_\nu^2 Z_t^T R_J(\phi_\nu) Z_t\right)$, or equivalently

$$Z_t^T\left(\hat{\xi}_t - X_t\beta_1 - X_{ts}\beta_2 + P_t\gamma - \theta\right) \sim N\left(0, \sigma_\nu^2 Z_t^T R_J(\phi_\nu) Z_t\right). \tag{19}$$

Note that this likelihood, denoted as $L(\xi_t)$, is specified over ξ_t, but the observed response variable is W_{tj}. Therefore, the likelihood for observed shares involves a non-linear transformation of variables from W_t to $\xi_t(W_t)$. Hence, we rewrite the likelihood $L(W_t) = L(\xi_t(W_t))\left|\frac{\partial\hat{\xi}_t}{\partial\hat{W}_t}\right|$ where the Jacobian is given by:

$$\left|\frac{\partial\hat{\xi}_t}{\partial\hat{W}_t}\right| = \begin{cases} \left[\int W_{tij}(1 - W_{tij})\, dF(\eta_i)\right]^{-1}, & \text{if } j = k \\ -\left[\int W_{tij}W_{tik}\, dF(\eta_i)\right]^{-1}, & \text{if } j \neq k. \end{cases} \tag{20}$$

[5] Unlike the time varying spatial effect ν_t, θ is a time-invariant spatial random effect. Accordingly it need not be instrumented. Analogous to Nevo's (2001) use of brand-specific dummies, θ_s can be estimated using location-specific dummy variables and the prior distribution $N\left(0, \sigma_\theta^2 R_J(\phi_\theta)\right)$.

[6] Inspection of the residual variograms evidence the exponential function fits the data well. Further, these residuals reveal little evidence of any anisotropy in spatial decay.

3.2 Supply-side Estimation

In the supply side model we seek to infer both the parameters in cost function and the non-zero Lagrangian multipliers. As noted in Section 3, we use observed demand market share $\hat{W}_{tj}$ to replace the expected market share W_{tj} in Equation (14). For the outlets at capacity constraint in period t, $\hat{W}_{tj} = \frac{K_j}{I}$ according to the Bertrand-Nash equilibrium. Using the price parameter estimates $\hat{\gamma}$ and $\hat{\sigma}_\gamma$ from the demand model, we compute ${\hat{\Omega}^{(t)}}^{-1}\hat{W}_t$ and impute it into the first order conditions:

$$\begin{aligned} P_t - \lambda_t + {\hat{\Omega}^{(t)}}^{-1}\hat{W}_t &= Y_t\beta_3 + \zeta_t, \\ W_{tj}\left(P_t, X_t, \theta_t\right) &= K_j/I \text{ if } \lambda_{tj} > 0 \end{aligned} \tag{21}$$

The markup for the outlet chain model is $\lambda_t - {\hat{\Omega}^{(t)}}^{-1}\hat{W}_t$ in which the subvector $\lambda_{1t} = 0$ for the outlets under capacity. For these outlets, $P_{1t} + \left[{\hat{\Omega}^{(t)}}^{-1}\hat{W}_t\right]_1 = Y_{1t}\beta_3 + \zeta_1 + e^c_{t,1}$. For the outlets at capacity, we have $\lambda_{2t} > 0$, or equivalently $P_{2t} + \left[{\hat{\Omega}^{(t)}}^{-1}\hat{W}_t\right]_2 > Y_{2t}\beta_3 + \zeta_2 + e^c_{t,2}$.[7] To use the full information available to infer the cost parameters, we supplement observed data from apartments below capacity with augmented data λ_{2t} for firms at or under capacity forming the following likelihood:

$$\left[\begin{array}{c} P_{1t} + \left[{\hat{\Omega}^{(t)}}^{-1}\hat{W}_t\right]_1 - Y_{1t}\beta_3 - \zeta_1 \\ P_{2t} + \left[{\hat{\Omega}^{(t)}}^{-1}\hat{W}_t\right]_2 - Y_{2t}\beta_3 - \zeta_2 - \lambda_{2t} \end{array}\right] \sim N\left(0, \sigma_c^2 \left[\begin{array}{cc} R_{11} & R_{12} \\ R_{21} & R_{22} \end{array}\right]\right), \tag{22}$$

where we partition the covariance matrix $\sigma_c^2 R\left(\psi_c\right)$ into four submatrices. To draw the augmented parameter λ_{2t} for use in equation (22) we use the following truncated distribution derived in Appendix I,

$$N\left(\lambda_{2t}|\tilde{P}_{2t} + R_{21}R_{11}^{-1}\tilde{P}_{1t}, \sigma_\zeta^2\left(R_{22} - R_{21}R_{11}^{-1}R_{12}\right)\right) I\left\{\lambda_{2t} > 0\right\}, \tag{23}$$

where $\tilde{P}_{kt} = P_{kt} + \left[{\hat{\Omega}^{(t)}}^{-1}\hat{W}_t\right]_k - \zeta_k - Y_{kt}\beta_3$, $k = 1, 2$.

As in the demand model, we assume an exponential correlation function (Banerjee et al. 2004) for the spatial random effects ζ and e^c_t. That is $R_J\left(\psi\right) = \exp\left(-\psi \cdot ||s - s'||\right)$. We assume the spatial random error ν_t in the indirect utility function is independent of the short-term supply-side error e^c_t. This does not mean demand and supply are independent; these are linked via the structural effect of observed effects and other factors on equilibrium prices. Note that instrumental variable

[7] Though the condition $P_{2t} + \left[{\hat{\Omega}^{(t)}}^{-1}\hat{W}_t\right]_2 > Y_{2t}\beta_3 + \zeta_2 + e^c_{t,2}$ might suggest that ζ_2 and $e^c_{t,2}$ are bounded from above, we note that P_{2t} is set after firms observe ζ_2 and $e^c_{t,2}$ and P_{2t} can be adjusted with ζ_2 and $e^c_{t,2}$. Hence this inequality does not place constraints on the cost shocks.

estimation on the supply side is not required because none of cost shifters in Y_t are correlated with e_t^c.

3.3 Model Inference

We use Bayesian inference to estimate the parameters in this model. The Bayesian estimation is accomplished by a Metropolis-within-Gibbs sampler in which σ_γ^2 is sampled using Metropolis-Hastings algorithm. The full conditional distributions for the sampling chain are provided in Appendix I.

3.4 Instrumental Variables

3.4.1 Selection of Instruments

In addition to price, the spatial shocks ν_{ts_j} may correlate with the observed attributes X_{tj} and X_{ts_j} (e.g., amenities may be more prevalent in more desirable areas). This potential correlation suggests the use of local attributes as instruments as in BLP (that is, $E\left(\nu_{ts_j}|X_{tj}, X_{ts_j}\right) = 0$) does not hold in the context of the outlet location problem. We consider instead as potential instruments Z_t, i) the proximity to work X_{ts_l} (from the same chain and from competitors), ii) attributes of distant outlets X_{tl} (such as clubhouse, tennis, swimming pool, gymnasium, and heat attributes from the same chain and from competitors), iii) distant cost shifters Y_{tl} (from the same chain and from competitors), and iv) lagged distant prices $P_{t-1,l}$ (from the same chain and from competitors). These are given respectively by

$$Z_{tj} = \begin{bmatrix} \sum_{\substack{l \neq j; l,j \in F_f \\ dist(l,j) > r}} X_{ts_l}, \sum_{\substack{l \neq j; l,j \notin F_f \\ dist(l,j) > r}} X_{ts_l}, \sum_{\substack{l \neq j; l,j \in F_f \\ dist(l,j) > r}} X_{tl}, \sum_{\substack{l \neq j; j \in F_f; l \notin F_f, \\ dist(l,j) > r}} X_{tl}, \\ \sum_{\substack{l \neq j; l,j \in F_f \\ dist(l,j) > r}} Y_{tl}, \sum_{\substack{l \neq j; l,j \notin F_f \\ dist(l,j) > r}} Y_{tl}, \sum_{\substack{l \neq j; l,j \in F_f \\ dist(l,j) > r}} P_{t-1,l}, \sum_{\substack{l \neq j; l,j \notin F_f \\ dist(l,j) > r}} P_{t-1,l} \end{bmatrix}, \tag{24}$$

where $l, j \notin F_f$ denotes that outlets l and j are not in the same outlet chain; $dist(l, j)$ is the distance between l and j; r is the cut-off distance for a competitor's inclusion in the summation. The considered instruments should be independent of spatial shocks and correlated with the endogenous prices. In light of this, we discuss our rationale for this set of instruments next.

Attributes as Instruments. Like BLP and Nevo (2001) we use attributes in other locations as instruments. However, in our context the unobserved spatial random effect ν_{ts_j} may be correlated with the local observed attributes X_{tj}, X_{ts_j} and cost shifter Y_{tj}. This correlation will decrease as outlet locations become far from each other. Beyond a distance r (which is called the "range" effect in spatial statistics) the correlation between the instruments and the spatial shock will approach zero. Moreover, if this cut-off distance is not sufficiently large, the attributes at other outlets will affect the local apartment's choice probability in the logit demand system and can therefore be expected to correlate with current prices. In sum, with the proper choice of r the distant attributes are uncorrelated with the errors but correlated with prices. The selection of the cut-off distance r must consider several factors. Selecting too large a distance will cause some outlets to have no instruments because no other outlets exist beyond a large distance. On the other hand, selecting too small a distance will lead to high spatial correlations between the competing apartment attributes and the local spatial shocks. Given the nature of competition differs for other locations within a chain and other locations within competing chains, we divide these instruments into the same-ownership group and different-ownership group.[8]

Prices as Instruments In our data, the correlation between lagged distant own outlet prices and price is 0.12 and the correlation between lag distant competitor outlet prices and price is -0.54 suggesting that lag distant prices may be good candidates for instruments. Our finding that P_{tj} correlates positively (negatively) with P_{tk} of outlet k in its own (a competing) chain is also confirmed in our simulation. These correlations can be explained as follows. First, lag distant prices are correlated with the long-term demand shock for these locales, θ_{s_j}. Second, the long-term demand shocks are correlated with prices. An increase in demand shocks for *competing* distant outlets makes those competing outlets more attractive. As a result, local firms must lower prices to compete. The price rule (10) suggests that a firm adjusts the prices of all its *own* outlets in the same direction. Hence, there is a positive correlation between the own chain and the local outlet. In this sense we argue that lagged distant prices for own and competing chains are correlated with current local price and reasonable choices for instruments.

We further reason that lagged distant prices are independent of local demand shocks (ν_t) because i) distant prices are correlated with distant demand shocks and ii) distant demand shocks are independent of local demand shocks and serially independent[9]. This logic rests on the assumption

[8] An analogous logic holds to justify our choice of cost shifters and distance to points of interest (schools and employers) as instruments.

[9] Using lagged distant price as an instrument affords advantages over using distant price as in Nevo (2001) when capacity constraints are present. Firms at capacity raise prices leading other firms to raise prices suggesting that

that the correlation of the demand shock does not extend as far as the effect of price on competing outlets. This assumption is likely to be true when the spatial decay is large (i.e., no spatial correlation) or the distance between the outlet and the competitive outlets used for instrumenting is sufficiently large.

3.4.2 Test of Instruments

To further explore the quality of our instruments, we first explored the orthogonality of the instruments by inspecting the plots of the marginal posterior distributions for $Z_t^T \nu_t$. We compute the 95% posterior predictive interval to assess whether it excludes 0 as a test of the orthogonality conditions. We find all the intervals contain 0. Note, however, this ignores the joint distribution of the orthogonality conditions. We therefore conducted an over-identifying test (J-test) of the quality of the instruments using the demand-side model. This test exploits the fact that the use of instruments that are correlated with the error will lead to biased parameter estimates – thereby leading to poor fit for the moment conditions of instruments that are not (i.e., $Z_t^T \nu_t \neq 0$). Table 1 presents the results of this analysis. In each period, the J-statistic does not differ significantly from zero, indicating the instruments are overall orthogonal to the model errors.

[Insert Table 1 Here]

To ascertain whether the instruments are correlated with the regressors, we computed the Shea's R_p^2 statistic. The results are reported in Table 1 and indicated a moderate correlation between the instruments and regressors, comparable to or higher than Thomadsen (2005). In light of the range restrictions on many of our instruments, the statistic indicates the instruments are reasonably well correlated with the regressors.

These tests also afford insights into our choice of the cutoff distance, r. Using a cutoff of 3.8 miles (which is chosen to be slightly greater than the 95% spatial decay (3.6 miles) interval estimated in the Results section), Table 1 indicates mean independence between our instruments and ν_{ts_j} using the J-test. Moreover, we find that smaller distances for r do not increase the correlation between the instruments and prices (the Shea's R_p^2 does not increase) suggesting little is to be gained by choosing a smaller distance.

current distant prices might not be good instruments. As lagged demand shocks do not affect whether capacity binds in the current period they are not afflicted by this consideration.

4 Spatial Prediction and Outlet Location Decision

Once estimates of the demand and supply model are obtained, it is possible to forecast how entry and capacity decisions at any given locale will affect the demand, prices and profits for the existing firms and the new entrant. Such information is useful to firms endeavoring to assess the profit potential of entering at various sites. The entering firm's decision problem consists of two steps: i) selecting the location and ii) setting optimal price and capacity conditioned on location. We can solve this decision problem by backward induction: if the location has been selected, the firm will set the price and capacity to maximize its profit. To achieve this aim, the firm must i) predict spatial random effects in demand and cost at the potential entry locations and then ii) compute the resulting equilibrium sales, prices and profits at these locations. We shall discuss each step in turn.

4.1 Spatial Prediction of Demand and Cost Effects: Bayesian Kriging

The first goal is to forecast random effects for long-term demand shock, θ_s, and cost shock, ζ_s, over space for any new location s in the future period T. This yields a map of latent spatial demand and cost that can be used to obtain insights into the nature of the market in which the outlets compete. To conserve space, we illustrate the spatial prediction for θ_s, as the same procedure applies to predict ζ_s. Using Bayesian kriging (Banerjee et al. 2004), we assume firms can estimate the distribution of the spatial random effects at the potential entry locations conditioned on the observed prices and sales at existing outlets. Suppose an entering firm considers building a single outlet at a new location $\tilde{s}_k \in (\tilde{s}_1, \ldots, \tilde{s}_n)$ in a future period T. To select a preferred location from potential locations $(\tilde{s}_1, \ldots, \tilde{s}_n)$, the firm would like to estimate the demand function at each of the considered locations $\tilde{s}_k$, $k = 1, \ldots, n$:

$$\tilde{Q}_{Tk} = I \times \tilde{W}_{Tk} \tag{25}$$

where

$$\begin{aligned} \tilde{W}_{Tk} &= \int\int \tilde{W}_{Tik} dF\left(\eta_i|\Theta\right) dF\left(\Theta|data\right) \text{ and} \qquad (26) \\ \tilde{W}_{Tik} &= \frac{e^{X_{Tk}\beta_1 + X_{T\tilde{s}_k}\beta_2 - P_{Tk}(\gamma+\eta_i) + \theta_{\tilde{s}_k} + \nu_{Ts_k}}}{\sum_{l=1}^{J} e^{X_{Tl}\beta_1 + X_{Ts_l}\beta_2 - P_{Tl}(\gamma+\eta_i) + \theta_{s_l} + \nu_{Ts_l}} + \sum_{m=1}^{n} e^{X_{Tm}\beta_1 + X_{T\tilde{s}_m}\beta_2 - P_{Tm}(\gamma+\eta_i) + \theta_{\tilde{s}_m} + \nu_{Ts_m}} + e^{M_T}}, \end{aligned}$$

and Θ represents all the parameters and random effects (e.g. β_1, β_2, γ, σ_γ^2, σ_ν^2, σ_θ^2, ϕ_θ and θ_{s_l}, $l = 1, \ldots, J$) in this demand system. Θ are sampled when we fit the model and the integrals can be approximated using Monte Carlo simulations for η_i. However, $\theta_{\tilde{s}_k}$, $k = 1, \ldots, n$ for the considered location are unknown as of yet. The estimation of the $\theta_{\tilde{s}_k}$ is spatial prediction. The entering firm is assumed to use Bayesian kriging to obtain the posterior distribution for latent demand $\theta_{\tilde{s}_k}$ and cost $\zeta_{\tilde{s}_k}$ at location $\tilde{s}_k$ and the demand function $\tilde{Q}_{Tk}$.

We assume that the location preference error θ_s follows a Gaussian random field. Thus, for any two locations s_1 and s_2, θ_{s_1} and θ_{s_2} have a bivariate normal distribution with the covariance calculated from the covariance function of the Gaussian process. With our choice of an exponential correlation function for the spatial random effects, $\exp(-\phi_\theta \cdot ||s - s'||)$, θ_{s_1} and θ_{s_2}'s covariance is given by $\sigma_\theta^2 \exp(-\phi_\theta \cdot ||s_1 - s_2||)$. Likewise, if there are J random variables $\theta_{s_1}, \ldots, \theta_{s_J}$ associated with location $s_1, \ldots, s_J$, the pairwise covariance of θ_{s_k} and θ_{s_j} is calculated as $\sigma_\theta^2 \exp(-\phi_\theta \cdot ||s_k - s_j||)$, which is also the (k, j)-th entry in covariance matrix of the multivariate normal distribution for $\theta_{s_1}, \ldots, \theta_{s_J}$. We denote this matrix as $\sigma_\theta^2 R_J$.

For the spatial prediction problem in our model the observed firms occupy the locations $(s_1, \ldots, s_J)$ and the entering firms select $(\tilde{s}_1, \ldots, \tilde{s}_n)$. The corresponding spatial random effects are $(\theta_{s_1}, \ldots, \theta_{s_J})$ and $(\theta_{\tilde{s}_1}, \ldots, \theta_{\tilde{s}_n})$, in which $(\theta_{s_1}, \ldots, \theta_{s_J})$ can be computed from equation (18) and the parameter draws from the Gibbs sampler. From the assumption of the spatial Gaussian process, $(\theta_{s_1}, \ldots, \theta_{s_J})$ and $(\theta_{\tilde{s}_1}, \ldots, \theta_{\tilde{s}_n})$ jointly have the following multivariate normal distribution:

$$(\theta_{s_1}, \ldots, \theta_{s_J}, \theta_{\tilde{s}_1}, \ldots, \theta_{\tilde{s}_n}) \sim N_{J+n}\left(0, \sigma_\theta^2 R_{J+n}\right) \tag{27}$$

in which $\sigma_\theta^2 R_{J+n}$ is a $(J+n) \times (J+n)$ covariance matrix where the covariance of θ_{s_j}(for current locations) and $\theta_{\tilde{s}_k}$(for future locations) is by definition $\sigma_\theta^2 \exp(-\phi_\theta \cdot ||\tilde{s}_k - s_j||)$.

In order to sample $(\theta_{\tilde{s}_1}, \ldots, \theta_{\tilde{s}_n})$ conditioning on the already sampled $(\theta_{s_1}, \ldots, \theta_{s_J})$, we partition $\sigma^2 R_{J+n}$ into four submatrices as follows:

$$\begin{bmatrix} \sigma^2 R_J & \sigma^2 R_{J,n} \\ \sigma^2 R_{J,n}^T & \sigma^2 R_n \end{bmatrix}$$

where $\sigma^2 R_{J,n}$ includes the covariances of θ_{s_j} and $\theta_{\tilde{s}_k}$, and $\sigma^2 R_n$ is the covariance matrix of $(\theta_{\tilde{s}_1}, \ldots, \theta_{\tilde{s}_n})$.

Conditioning on $\theta \triangleq (\theta_{s_1}, \ldots, \theta_{s_J})$, $\tilde{\theta} \triangleq (\theta_{\tilde{s}_1}, \ldots, \theta_{\tilde{s}_n})$ has the following distribution:

$$\tilde{\theta} \sim N_n\left(R_{J,n}^T R_J^{-1} \theta, \sigma^2 \left[R_n - R_{J,n}^T R_J^{-1} R_{J,n}\right]\right) \tag{28}$$

This is a n-variate conditional normal distribution with mean vector $R_{J,n}^T R_J^{-1}\theta$ and covariance matrix $\sigma^2 \left[R_n - R_{J,n}^T R_J^{-1} R_{J,n}\right]$. Sampling $(\theta_{\tilde{s}_1}, \ldots, \theta_{\tilde{s}_n})$ from this distribution is called Bayesian spatial prediction or kriging.

Equation (28) also affords insights into the effect of spatial covariance on estimated demand. When spatial covariance is ignored ($R_{J+n}(\phi) = I_{J+n}$), the conditional spatial random effects are attenuated in expectation to zero ($E\left(\tilde{\theta}|\theta\right) = R_{J,n}^T R_J^{-1}\theta \neq 0$ if $R_{J+n}(\phi) \neq I_{J+n}$). As prices rise with demand (because firms raise prices to capitalize on increased demand), an improper accounting of these effects countervails the fall in market share with price so that their omission leads to downward bias in profits.

4.2 Predicting Price and Profit

Once predictions of demand and cost effects are obtained, the second step of the outlet location problem is to predict profits and prices in new locales. This problem can be formulated as:

$$\max \Pi_{Tk} = (P_{Tk} - \tilde{c}_{Tk})\,\tilde{Q}_{Tk}, \tag{29}$$

where

$$\tilde{c}_{Tk} = \int \left\{Y_k\beta_3 + \zeta_{\tilde{s}_k} + e^c_{T\tilde{s}_k}\right\} dF(\beta_3, \zeta_{\tilde{s}_k}, e^c_{T\tilde{s}_k}|data). \tag{30}$$

There is no capacity constraint as the firm can build to demand.[10] Hence, the entering firm maximizes its profit with respect to P_{Tk} and selects the capacity $\tilde{K}_{Tk} = \tilde{Q}_{Tk}(P_T, X_T, \Theta_T)$. The first order condition is:

$$\frac{\partial \tilde{\Pi}_{Tk}}{\partial P_{Tk}} = (P^*_{Tk} - \tilde{c}_{Tk})\,\frac{\partial \tilde{W}_{Tk}(P^*_{Tk}, P_T)}{\partial P_{Tk}} + \tilde{W}_{Tk}(P^*_{Tk}, P_T) = 0 \Rightarrow$$

$$(P^*_{Tk} - \tilde{c}_{Tk}) \int\int -(\gamma + \eta_i)\tilde{W}_{tik}\left(1 - \tilde{W}_{tik}\right) dF(\eta_i|\Theta)\, dF(\Theta|data) + \tilde{W}_{Tk} = 0. \tag{31}$$

The existing outlets will adjust their prices accordingly, knowing the location selection $\tilde{s}_k$ of the entering firm. Their strategic pricing problem is:

$$\max \Pi_{Tf} = \sum_{j \in F_f} (P_{Tj} - \tilde{c}_{Tj})\,\tilde{Q}_{Tj} \text{ s.t. } \tilde{Q}_{Tj} \leq K_j \tag{32}$$

[10] It should be noted that there is a marginal construction cost associated with the addition of each unit of capacity. However, all the construction costs become sunk once the apartment building is built. Our analysis proceeds on the assumption that the net present value of discounted profits exceeds the total construction cost.

Ideally the optimal price of firm f is determined by the following Kuhn-Tucker condition:

$$\frac{\partial \Pi_{Tf}}{\partial P_{Tl}} = \sum_{j \in F_f} \left(P_{Tj} - \tilde{c}_{Tj}\right) \frac{\partial \tilde{W}_{Tj}}{\partial P_{Tl}} + \tilde{W}_{Tl} - \sum_{m \in F_f} \lambda_m \frac{\partial \tilde{W}_{Tm}}{\partial P_{Tl}} = 0; l \in F_f \tag{33}$$

$$\begin{array}{c} I \cdot \tilde{W}_{Tm} = K_m, \text{ iff } \lambda_m > 0 \\ \text{and } I \cdot \tilde{W}_{Tm} < K_m, \text{ iff } \lambda_m = 0 \end{array}. \tag{34}$$

The corresponding demand $\tilde{Q}^*_{Tl} = Q_{Tl}\left(P^*_{Tl}, P^*_{Tk}, P^*_{T,-l}\right)$ may exceed the capacity K_l. The implementation of the optimization problem involves two steps: with a certain set of interim prices P_{Tl}, P_{Tk}, the demand $\tilde{Q}_{Tl}$ and $\tilde{Q}_{Tk}$ are calculated; if $\tilde{Q}_{Tl} \geq K_l$, the corresponding P_{Tl} is solved from the binding constraint in (34) in the next step of the optimization; and if $\tilde{Q}_{Tl} < K_l$, the corresponding P_{Tl} is solved from (33) in the next step.

An important consideration is the uniqueness of prices in this equilibrium as multiple price equilibria would imply different profit outcomes for each equilibrium. In the Technical Appendix II (available online), we prove the existence and uniqueness of the equilibrium prices in equation (33) by constructing a contraction mapping on bounded set.

Equation (31) for the entering firm and (33) for the existing firms constitute the strategic optimization problem that solves the new P^*_{Tl}, P^*_{Tk}, $\tilde{Q}^*_{Tk}$ and the profit Π^*_{Tf}, Π^*_{Tk}. Π^*_{Tk} is a function of location $\tilde{s}_k$. Optimizing Π^*_{Tk} with respect to the location $\tilde{s}_k$ solves the optimal market entry problem. We simulate demand for the entrant using a modal configuration of attributes (across the existing outlets).

5 Data

To estimate our model we use panel data on apartment demand and prices. Apartments are a desirable category for illustrating the model because the number of outlets is sufficiently small to make estimation feasible but sufficiently large to obtain relatively reliable estimates of spatial effects. Moreover, unlike many previous applications of spatial demand models in economics, latent demand for apartments can not be estimated as a function of the underlying observed spatial distribution of the population as the population itself is endogenous. We use data from Roanoke, Virginia for our analysis. This market shows good spatial coverage of apartments, little variability in supply, and a long time series of prices and vacancy rates are available for these markets. These characteristics make them ideal for our analysis.

The data for this study are provided by Real Data of Charlotte, North Carolina and are detailed at www.aptindex.com. Real Data conducts an annual survey of apartments in a given market. The survey data consist of apartment attributes (such as whether the apartment has tennis courts, whether it has a pool, the age of the complex, etc.), addresses (which we convert to latitude and longitude), and prices. The Roanoke data cover 60 apartment complexes managed by 25 different firms, covering 7 years between 1999 and 2005. Figure 1 depicts the location and occupancy levels of the apartments in 2005; the shortest bar corresponds to 50 rented units and the largest apartment corresponds to 426 rented units. The average apartment capacity of these apartments is 152 units. Eight apartments are missing several years of data and are thus excluded from subsequent analysis.

[Insert Figure 1 Here]

The apartment data is supplemented by two other data sources. First, we use the 2000 census data to attain the locations of schools and the market size, I. The census data at 'www.census.gov/geo/www/tiger' contains the latitude and longitude of the Roanoke schools. From these data we computed the distance to the nearest school for use as an observed spatial covariate in our analysis. Other census data at 'www.fedstats.gov/qf/states/51' reports the number of non-home owning households. We use this to determine I (for the Roanoke area, this was 29,489 households in 2000).[11] Second, we determined the major employers in Roanoke from the Roanoke County Department of Economic Development (www.yesroanoke.com). For the largest employers with a single central location, we computed the mean distance from each complex to the major employers. Table 2 summarizes the variables we use in our analysis:

[Insert Table 2 Here]

To construct a price index from these prices for various apartment sizes, we use the price of two bedroom apartments (which we report in Table 2). We choose this simple approach for a couple of reasons. First, to construct a measure that considers all apartments, we would need to create an index weight – and these weights may embed capacities, sales or other factors that 'contaminate' the pricing measure. Second, all of the apartments have two bedroom units, and the two bedroom

[11] Our results are not particulary sensitive to reasonable changes in this value.

size is always the modal size (many apartments have only two bedroom units). Hence, it is the most representative. When an apartment has multiple prices for the two bedroom units, we select the median value (no information is available on the distribution of prices for a given apartment size at a complex). During the interval of the data, average apartment prices in Roanoke increased from $497 to $570 per unit though they remained constant the last three years. In the analysis, we adjust prices for inflation using CPI figures from the Bureau of Labor Statistics. Figure 2 presents a contour map of 2005 rents in nominal dollars.

[Insert Figure 2 Here]

From Figure 2 we observe that the distribution of equilibrium prices is highly irregular, with the highest prices observed both near the downtown and on the periphery of town. A band of lower prices snakes from northwest to southeast. Given that prices change non-monotonically with distance from the city center, it is desirable to capture random spatial effects rather than relying upon the more commonly used approach wherein demand is a linear function of distance to a population centroid such as the center of town. The proposed spatial random effects approach can yield highly irregular demand and price surfaces. The spatial distribution of prices in Figure 2 also suggests there exists spatial covariance in the data.

Vacancy rates averaged roughly 6% with little change over time. In addition, total annual market capacity was fairly constant over time with a mean of 8985 units and a standard deviation of 337 units. No apartments were being constructed as of 2005. This suggests it is appropriate to model the sub-game of prices in our data conditioned on apartment locations and capacities. Figure 3 depicts a contour map of vacancy rates. Vacancy rates tend to be highest on the west side of town.

[Insert Figure 3 Here]

6 Results

6.1 Simulation Results

To i) show how the omission of spatial covariance and capacity constraints biases model estimates and ii) show that our model can recover the underlying parameters, we conduct a simulation using the approach described in Appendix III online. Using these simulated data we estimate three models. In model one, we include spatial covariance and capacity constraints. In model two, we omit spatial covariance. In model three, we omit capacity constraints. The Gibbs sampling chain proceed for 10000 iterations in each model though they appear to converge well before then (within 500 iterations). We discarded the first 2000 draws to ensure convergence. Priors are set to be as non-informative as possible. We use all available attributes and prices from preceding periods as instruments. Below, we report the parameter estimates and the ability of three models to recover the parameters.

[Insert Table 3 Here]

6.1.1 Full Model

As indicated in Table 3, the 95% posterior predictive interval for model 1 contains the true parameter values for all of the parameters in our properly specified model indicating that the model can recover the data generating mechanism. Of particular interest is the recovery of the estimate for the highest Kuhn-Tucker multiplier. This parameter captures the marginal cost of the constraint to the firm; in this case chain 39 in year 8 could gain $131 in monthly profit if it could add one apartment.

6.1.2 No Spatial Effects

Contrasting the full model to one wherein we omit spatial correlation (model 2), we observe the predicted downward bias in median price effects. The magnitude of this effect in this parameter is 20%. This bias is a consequence of how covariation in the spatial errors can exacerbate small sample bias in GMM and IV estimation (Altonji and Segal 1996, Buse and Moazzami 1991). When these correlations are high, bias is present even in fairly large samples (Altonji and Segal 1996).

We also observe a downward bias of about 20% in the estimate for the intercept of marginal cost, β_{31}. The cause of this downward bias in estimated marginal costs can be seen by inspecting Equation (14). The downward bias in the estimate for price elasticity γ lowers the expression on the left hand side of Equation (14). To maintain this equality, estimates for the right hand side must also be lower leading to a downward bias in the estimates for marginal cost. We also note that there is a substantial decrease in the demand-side log marginal likelihood from -27.4 to -43.9.[12]

[12] The ratio of the marginal likelihood (exponential of the *log marginal likelihood*) is equivalent to Bayes factor

The log marginal likelihood on the supply side decreases from -1093 to -1130.

6.1.3 No Capacity Constraints

Contrasting the full model to one wherein we omit capacity constraints, we note that cost estimates are biased upwards. Firms raise prices in the presence of capacity constraints to the point where demand is equal to capacity because lower prices yields lower per unit revenue with no attendant increase in demand. The observation of higher prices at a given level of demand leads to inferences of higher costs, consistent with Equation (14). Further, the variance of the fixed and time varying spatial cost effects σ_ζ and σ_e are overestimated by nearly a factor of 2 as a result of the additional error introduced by ignoring the capacity constraints. The spatial decay is overestimated by 150%. Reflecting these biases, the log-marginal likelihood decreases considerably from -1093 to -1276.

In sum, the simulation data evidence i) that our model recovers parameters well and ii) biases in parameter estimates that arise from ignoring spatial covariance and capacity constraints are in the predicted direction. Moreover, omitting spatial effects and capacity constraints has a substantial impact on model fit.

6.2 Roanoke Results

We next apply the data detailed in Section 5 to estimate our model of outlet demand and pricing. We ran the sampling chain for 20000 iterations. Inspection of the sequence of draws indicates good convergence after about 500 iterations though we discarded the first 2000 iterations. Moreover, little autocorrelation in the draws was evident. In addition to estimating the full model, we provide two benchmarks; no spatial correlation and no capacity constraints. Comparison of these models to the full model yields insights into i) the magnitude of parameter biases that can arise when capacity and spatial covariance in demand and prices are ignored and ii) the degree to which spatial covariance improve model performance.

6.2.1 Parameter Estimates

Table 4 presents the results of our estimation. The full model is the best fitting model and its improvement over the model with no spatial covariance is sizable (the improvement in the log

with noninformative priors and we therefore use this statistic for model comparison. Log marginal likelihoods are computed as in Gelfand and Dey (1994).

marginal likelihood is 18 on the demand-side and 22 on the supply-side). This affords evidence that spatial covariance matters in practice. The improvement over a model with not capacity constraints is even more substantial leading to a 141 point gain in the log marginal likelihood.

[Insert Table 4 Here]

Demand-side Estimates. All attributes but the addition of a gym play a significant role in apartment choice. The low effect of gym may arise from many competing options including work and office gyms as well as home exercise equipment. Among the attributes, pool and tennis courts each play the greatest role in apartment choice. The 95% posterior predictive interval for distance to schools includes zero suggesting this effect is negligible.[13] The small effect may reflect few school age children among apartment dwellers or the local school zoning policies. The effective range of the median spatial decay, wherein 95% of the spatial effect has decayed is given by $3/\phi_\theta$ (Banerjee et al. 2004), or 3.6 miles. This suggests that the demand-side spatial covariance is sizable, as the maximum distance between apartments is 9.6 miles. The price parameter is positive, indicating that an increase in price lowers the likelihood of apartment choice (as this enters our likelihood function with a negative sign). Consistent with our previous findings, price parameter is biased toward zero when one ignores spatial covariance and capacity constraints; the bias is roughly 10%.

Supply-side Estimates. Table 4 indicates that heat increases the variable costs of the apartment about \$70 per month per unit in 1999 dollars and that newer apartments have higher operating costs, perhaps due to greater amenities. The highest median Kuhn-Tucker multiplier is \$88 per month per unit for apartment 37 in year 2004. Among apartments at capacity, this apartment had the second lowest costs. Because this apartment is so efficient, its capacity constraint were especially costly. We also note that apartments with high rents tended to have high costs of capacity constraints.

According to a 1998 survey of apartment managers conducted by the Institute of Real Estate Management (IREM), annual operating expenses for apartments average 42% of revenue and these costs include administrative expenses, operating expenses, maintenance expenses, tax/insurance, and payroll and amenities. For the average rent of \$485 in our data (expressed in 1999 dollars), this implies our estimates for variable costs should be in the neighborhood of \$204 per apartment in

[13] When estimating the model we noted that the correlation between distance to schools and distance to employers was high so we omitted the latter as it had less explanatory power.

1999 dollars. Averaging our marginal cost estimates across apartments yields \$233 in 1999 dollars, quite close to the \$204 implied by the IREM survey.

The spatial effects on the supply-side (prices), with an effective range of 1.1 miles, are smaller than those on the demand-side. We conjecture that costs are a more “global” variable in the sense that all apartments likely face a similar cost of capital, utilities, labor and supplies. On the demand-side, however, certain regions are more desirable than others leading to higher spatial correlation.

As expected, cost estimates are biased downward when spatial covariance is ignored and the cost error variance, increases when capacity constraints are ignored. We do not observe an upward bias in costs for the no capacity constraint model perhaps due to the limited number of apartments are constrained in our data. Consistent with the simulation findings, the no capacity constraint cost model has poorer fit as indicated by higher estimates for ζ and σ_c. These parameters are respectively 16% and 30% too high when capacity is ignored.

6.2.2 Latent Spatial Effects

Using the median of the samples of the spatial random effects, we create a contour plots of spatial random demand θ_{s_j} and cost effects ζ_{s_j} and offer a discussion of these results. Figure 4 plots the θ_{s_j} and then interpolates latent demand between these observations via triangle based cubic interpolation.[14] This yields a map of latent demand.

[Insert Figure 4 Here]

The Figure indicates several modes of high latent demand; including just west of downtown and a more prominent mode southwest of downtown. It is interesting to compare Figure 4 which computes the latent spatial demand effects with Figure 2 which depicts vacancy. The highest demand region southwest of town not only has high latent demand but also higher prices. Taken together, these factors would imply that this modal location for demand is an ideal spot to locate a new complex.

Yet selecting an optimal outlet locale must also assess the effects of competition and costs. For example, Figure 3 indicates that the mode in latent demand southwest of downtown also

[14] The Figure interpolates latent demand for locations where there are no apartments based on our observed estimates for θ as well as kriged θ in a 10*10 grid.

corresponds with higher vacancy rates due a surfeit of apartments. Hence, the problem of outlet location is a complex calculus involving an array of countervailing factors. It is these tradeoffs we seek to explore more precisely in the next section.

7 Managerial Implications

One can use our approach to engage policy experiments pertaining to the selection of a new outlet location as it structurally links prices to outlet entry. For example, a firm could compare across available properties the effect of locating an additional outlet on demand, prices and profits at i) the additional outlet and ii) other outlets in the chain. In our analysis we compare the desirability of these entry options subject to the caveat that no other outlets enter. However, even in the context of competitive response, one could simulate the effect of various competitive location responses to the firm's choices of next outlet location. This suggests that many scenarios could be played to simulate entry effects.

One might conjecture that firms already occupy the optimal locations, so that the policy simulation is of little value. We think this is unlikely primarily because the ebb and flow of persons into the market and changes in the attributes of various locales (e.g., new roads) over the years likely renders the extant distribution of existing outlets sub-optimal. It is therefore likely to be useful to conjecture what the next best location may be.

Using the procedure discussed in Section 4.2, we assess the effect of an additional apartment on equilibrium demand, prices and profits. We begin by creating a 10 by 10 grid of potential apartment entry locations within the convex hull defined by extant apartment locales in Roanoke. For each location on the 100 point grid we compute equilibrium profits associated with an entry at that location. This computation assumes a modal apartment design for the new complex, that is, using the modal feature set in the data. We execute this procedure twice; once for our full model and once for a model that omits spatial covariance and capacity constraints. This enables us to contrast the recommendation of each model. Figure 5 depicts the predicted equilibrium profits at these locations.

[Insert Figure 5 Here]

The top half of Figure 5 portrays the equilibrium profits for the various entry options using the full model. The areas of the solid circles correspond to predicted monthly profits, with a maximum of $\$61,427$; a median of $\$37,025$; and a minimum of $\$20,356$. The bottom half of Figure 5 depicts predicted profits for the various entry options using the model with no spatial effects. The predicted profits from this model range from $\$38,136$ down to $\$35,572$. The median predicted profit is $\$36,971$. Most strikingly, the constrained model evidences a lack of variation in predicted profits as it ignores unobserved information regarding latent demand and costs from extant apartments to forecast demand and costs at new locales. Instead, it uses only observed spatial effects such as distance to the nearest school and these observed effects appear to be quite small relative to the unobserved spatial variation. The standard deviation in predicted profits is 10.8 times larger for the spatial covariance model. Using this model, one would inadvertently conclude some of the lower profit locations would yield a good profit. Further, the Figure indicates several "false modes", wherein the full model predicts small profits.

The open circles in the top and bottom half of Figure 5 enclose the locales with the highest predicted profit in each of the respective models. For the full model, the highest predicted profit corresponds with a lesser mode in latent demand just west of downtown evidenced in Figure 4. The no spatial variation demand model recommends a locale in the far southeastern part of Roanoke. Were one to adopt the recommendation of the model without spatial covariance, expected profits would be $\$37,062$ (which represents the predicted profits from the full model using the recommended locale from the constrained model). Using the recommendation from the spatial model increases expected profits to $\$61,427$ or 66%. When one considers that these estimates are cash flows and that firms expand into many different cities, the profit implications of our model could be considerable.

[Insert Figure 6 Here]

The second-highest mode in profits is co-located with the highest mode in latent demand discussed in Section 6.2.2. To provide more intuition regarding why the lesser mode in latent demand west of town is optimal and the larger mode in latent demand southwest of town is not, Figure 6 depicts the optimal locale superimposed on the spatial distributions of historical rental prices, historical vacancy rates, latent costs and latent demands. In these distributions, a darker shade corresponds with a lower number. Apartment locations are indicated in the Figure by black squares and the optimal entry locale is denoted by a white circle. The optimal location results because i) latent demand is high, ii) latent costs are low and iii) vacancy rates are low. Though the historical rental prices are low, this is in part offset by lower costs suggesting high margins are possible even in the face of lower rents. Further, there is only one competing complex in this area. In contrast, the second highest mode in profits and highest mode in latent demand corresponds with i) higher latent costs, ii) higher vacancy rates and iii) more competing chains. Hence, the presence of competition affects the recommendation for optimal location. We contend that widely employed tools for site location often neglect this aspect of the site location problem (Buckner 2004).

8 Conclusions

In this manuscript we address the problem of the firm's outlet location problem. A necessary step in this process is to solve the sub-game problem of firm pricing conditioned on firm locations. Firms can then use this model to simulate the effect of locating an incremental outlet on equilibrium prices, demand and profits. Our model applies these concepts in the context of apartment data and affords recommendations about the next best location for entry.

Our work extends prior research in a number of respects. Unlike gravity models, we explicitly and structurally consider competitive response in prices. Unlike analytical models, our approach is data-focused to enable insights into data-driven decisions in a number of different marketing contexts. We extend structural model of the subgame of outlet location and prices to include, among other things, spatial correlation and capacity constraints. It is desirable to consider spatial correlation for unobserved random demand and costs effects for several reasons. First, the omission of spatial covariation in demand leads to downward biased estimates of price effects and marginal costs in the small sample sizes common in the outlet location problem. Second, spatial demand effects are interesting in their own right, leading to a managerially informative map of latent demand. Neglecting these effects obscures considerable spatial demand variation inherent in our data. It is also desirable to consider capacity constraints. A proper accounting of these effects mitigates a upward bias in costs and the Kuhn-Tucker multipliers provide an estimate of the cost of the capacity constraint. Presumably, firms with higher costs would be more inclined to consider additional capacity. In addition, failure to consider capacity constraints and spatial effects leads to suboptimal recommendations for firm location.

To achieve these aims we integrate Bayesian spatial statistics with structural models of competition in a model of outlet demand and pricing. Simulations indicate that the resulting model recovers parameters well and that ignoring spatial effects and capacity constraints leads to biased parameter estimates. We then apply this model to a novel apartment data set that includes prices, demand and capacities over a 6 year period. We find that the inclusion of spatial covariance in demand improves model fit and that omitting spatial effects and capacity constraints lead to biased parameter estimates as predicted.

Given we seek to assess the policy implications of outlet location, we use a spatial kriging approach coupled with our supply-side model to explore the impact of locating an additional outlet at potential sites of interest to a firm (or more systematically upon a grid). These simulations reveal i) the best entry locations relate to high latent spatial demand and a dearth of nearby outlets, ii) accommodating capacity and unobserved spatial effects in policy simulations improves the profitability of recommended entry locales by 66% in our data, and iii) ignoring spatial covariance leads to little variation in predicted profits, understating the actual standard deviation in predicted profits across space by a factor of 11.

A number of limitations exist that also represent opportunities for future research.

- We consider the pricing sub-game only and do not model the outlet location game. We do this because the sub-game is an important problem in its own right and is useful for policy simulations over the intermediate term. An important extension would be to consider the entry problem as well. We think this would be a difficult extension as it involves a dynamic program to solve for sequential entry and prices and it is likely that solutions to this problem would not be unique. Moreover, the distribution of latent demand can render such equilibria obsolete after a few years.

- Many outlets sell multiple goods or have products that appeal to different markets (e.g. supermarkets or medical centers). It would be desirable to ascertain whether competition across goods leads to different outcomes for location and pricing than in the indexed single good case modeled herein.

- We presume attributes such as pool and tennis are exogenous. It would be desirable to model not only the location of an outlet, but also its optimal design. We believe that this exercise in combinatorial optimization would prove challenging, especially with respect to documenting the uniqueness of these equilibria.

- Our model is of limited applicability when alternative channels such as the Internet comprise a significant portion of demand. Multi-channel models (Ansari, Mela and Neslin 2007) could be overlaid with the spatial models to develop unique insights into this setting.

- In Appendix II (available online), we establish the uniqueness of prices on the supply side under the condition that price sensitivity is negative. Given the supply-side model is therefore a function, this result suggests it may be possible to develop full information likelihood-based inferential technique for supply and demand. Such an approach could lead to gains in efficiency over extant estimation approaches and yield more desirable small sample properties.

- Finally, it would be interesting to consider capacity a strategic variable. As we observe only one complex change capacity in our data it is an ideal setting to explore Bertrand-Nash competition; however we think it desirable to extend this research to the context of Cournot competition. Of note, the Bertrand and Cournot pricing outcomes align in a two stage game wherein capacity is set in stage 1 and prices are realized conditioned on capacity in stage 2 (Kreps and Scheinkman 1983). Given we model prices in a subgame conditioned on capacity constraints, this suggests the predicted prices in our model may be equivalent to the Cournot pricing outcomes.[15]

In sum, the approach we develop incorporates spatial effects into a model of outlet competition and affords insights into the role of unobserved spatial effects on outlet pricing and demand. We hope that our research will spark some subsequent research related to these and other remaining issues.

[15] We thank an anonymous reviewer for this observation.

Appendix I: Sampler in Section 3.3

Demand Model

Step 1. Sample $\left(\tilde{\theta}, \beta_1, \beta_2, \gamma\right)$.

The location-specific effects for an outlet are given by $\tilde{\theta} \sim N\left(X\alpha_1 + X_s\alpha_2, \sigma_\theta^2 R_J(\phi_\theta; d)\right)$, where X and X_s are time-invariant attributes. Note that , $\theta = \tilde{\theta} - X\alpha_1 - X_s\alpha_2$. Let $X_{tA} = [I_J, X_t, X_{ts}, P_t]$ and $\theta_A = \left(\tilde{\theta}, \beta_1, \beta_2, \gamma\right)$. θ_A has the multivariate normal prior $N(\mu_0, \Sigma_0)$, where

$$\mu_0 = (X\alpha_1 + X_s\alpha_2, \beta_{10}, \beta_{20}, \gamma_0)^T \text{ and } \Sigma_0 = \begin{bmatrix} \sigma_\theta^2 R_J(\phi_\theta) & 0 \\ 0 & \Sigma_0^{\beta,\gamma} \end{bmatrix}$$

The prior is conjugate to the likelihood. The full conditional for θ_A is $N(\mu_A, \Sigma_A)$, where

$$\Sigma_A = \left\{ \frac{1}{\sigma_\nu^2} \left(\sum_{t=1}^{T} X_{tA}^T Z_t \right) \left(\sum_{t=1}^{T} Z_t^T R_J(\phi_\nu) Z_t \right)^{-1} \sum_{t=1}^{T} Z_t^T X_{tA} + \Sigma_0^{-1} \right\}^{-1}; \tag{A-1}$$

$$\mu_A = \Sigma_A \left\{ \frac{1}{\sigma_\nu^2} \left(\sum_{t=1}^{T} X_{tA}^T Z_t \right) \left(\sum_{t=1}^{T} Z_t^T R_J(\phi_\nu) Z_t \right)^{-1} \sum_{t=1}^{T} Z_t^T \hat{\xi}_t + \Sigma_0^{-1}\mu_0 \right\}. \tag{A-2}$$

Step 2. Sample α_1, α_2 and θ.

Let $\alpha_A = (\alpha_1, \alpha_2)$ and $X_A = (X, X_s)$. Given $\tilde{\theta}$, σ_θ^2, ϕ_θ and the prior $N(\mu_0^\alpha, \Sigma_0^\alpha)$ for the full conditional distribution from the posterior for α_1 and α_2 is

$$N\left\{ \left(\frac{1}{\sigma_\theta^2} X_A^T R_J^{-1}(\phi_\theta) X_A + \Sigma_0^{\alpha^{-1}} \right)^{-1} \left(\frac{1}{\sigma_\theta^2} X_A^T R_J^{-1}(\phi_\theta) \tilde{\theta} + \Sigma_0^{\alpha^{-1}} \mu_0^\alpha \right), \left(\frac{1}{\sigma_\theta^2} X_A^T R_J^{-1}(\phi_\theta) X_A + \Sigma_0^{\alpha^{-1}} \right)^{-1} \right\}. \tag{A-3}$$

In our Roanoke data analysis, we use independent and very disperse priors for α_1, α_2, β_1, and β_2: $N\left(0, 10^8\right)$.

Step 3. Sample σ_γ and $\hat{\xi}_t$: Metropolis-Hastings Algorithm.

Suppose σ_γ has a truncated normal prior $N\left(\sigma_\gamma | 0, \sigma_0^2\right) I\{\sigma_\gamma > 0\}$. Use a truncated normal $N\left(\sigma_\gamma | \mu_p, \sigma_p^2\right) I\{\sigma_\gamma > 0\}$ to generate $\sigma_\gamma^{(n)}$. After recalculating $\hat{\xi}_t^{(n)}$ conditioned on $\sigma_\gamma^{(n)}$using contraction a mapping as indicated in Section 3.1, the acceptance probability for σ_γ and $\hat{\xi}_t$ is given as:

$$\min\left\{\frac{\prod_{t=1}^{T} N\left(Z_t^T\hat{\xi}_t^{(n)}|Z_t^T X_{tA}\theta,\sigma_\nu^2 Z_t^T R_J(\phi_\nu)Z_t\right)\left|\frac{\partial\hat{\xi}_t^{(n)}}{\partial\hat{W}_t}\right| N\left(\sigma_\gamma^{(n)}|0,\sigma_0^2\right)N\left(\sigma_\gamma|\mu_p,\sigma_p^2\right)}{\prod_{t=1}^{T} N\left(Z_t^T\hat{\xi}_t|Z_t^T X_{tA}\theta,\sigma_\nu^2 Z_t^T R_J(\phi_\nu)Z_t\right)\left|\frac{\partial\hat{\xi}_t}{\partial\hat{W}_t}\right| N\left(\sigma_\gamma|0,\sigma_0^2\right)N\left(\sigma_\gamma^{(n)}|\mu_p,\sigma_p^2\right)},1\right\}. \tag{A-4}$$

Note $\hat{\xi}_t$ is deterministic (obtained by a contraction mapping and Monte Carlo integration) given σ_γ.

Step 4. Sample σ_θ^2 and σ_ν^2.

We use the conjugate $Inverse - Gamma\,(1,1)$ priors for σ_ν^2 and σ_θ^2. The posterior distributions are also inverse-gamma.

Step 5. Sample ϕ_θ and ϕ_ν: discrete sampler.

Assume ϕ_ν (or ϕ_θ) can only take n values: $(\phi_1,\ldots,\phi_n)$. For each ϕ_ν (or ϕ_θ), calculate the posterior probability

$$\left|Z_t^T R_J(\phi_i) Z_t\right|^{-\frac{1}{2}}\exp\left\{-\frac{1}{2\sigma_\nu^2}\left(\sum_{t=1}^{T}\left(\hat{\xi}_t - X_{tA}\theta_A\right)^T Z_t\right)\left(\sum_{t=1}^{T} Z_t^T R_J(\phi_\nu) Z_t\right)^{-1}\sum_{t=1}^{T} Z_t^T\left(\hat{\xi}_t - X_{tA}\theta_A\right)\right\}$$

$$\left(\text{or } \left|R_J(\phi_i)\right|^{-\frac{1}{2}}\exp\left\{-\frac{1}{2\sigma_\theta^2}\theta^T R_J^{-1}(\phi_i)\,\theta\right\}\right).$$

Sample ϕ with replacement from $(\phi_1,\ldots,\phi_n)$.

Cost Model

Step 1. Sample β_3 and ζ.

Let $Y_{tA} = (I_J, Y_t)$. Let $\zeta_A = (\zeta,\beta_3)$. ζ_A has the multivariate normal prior $N(\mu_0,\Sigma_0)$, where

$$\mu_0 = (0,\beta_3)^T \text{ and } \Sigma_0 = \begin{bmatrix} \sigma_\theta^2 R_J(\phi_\theta) & \\ & \Sigma_0^{\beta_3} \end{bmatrix}$$

Given λ_{2t} and γ (γ is sampled from demand model), the likelihood is

$$\exp\left(-\frac{1}{2\sigma_c^2}\sum_{t=1}^{T}\left(P_t - B_t(\lambda_t,\gamma) - Y_{tA}\zeta_A\right)^T R_J^{-1}(\psi_c)\left(P_t - B_t(\lambda_t,\gamma) - Y_{tA}\zeta_A\right)\right). \tag{A-5}$$

where $B_t(\lambda_t, \gamma) = \lambda_t - \hat{\Omega}^{(t)^{-1}} \hat{W}_t$ ($\lambda_t = (\lambda_{1t}, \lambda_{2t})^T$ with $\lambda_{1t} = 0$ and $\lambda_{2t} > 0$). The full conditional for ζ_A is $N(\mu_A, \Sigma_A)$, where

$$\Sigma_A = \left(\frac{1}{\sigma_\zeta^2} \sum_{t=1}^{T} Y_{tA}^T R_J^{-1}(\psi_c) Y_{tA} + \Sigma_0^{-1} \right)^{-1} \text{ and } \tag{A-6}$$

$$\mu_A = \Sigma_A \left(\frac{1}{\sigma_\zeta^2} \sum_{t=1}^{T} Y_{tA}^T R_J^{-1}(\psi_c)(P_t - B_t(\lambda_t, \gamma)) + \Sigma_0^{-1} \mu_0 \right). \tag{A-7}$$

In our Roanoke data analysis, we use independent and very disperse priors for β_3: $N\left(0, 10^8\right)$.

Step 2. Sample λ_{2t}: data augmentation.

Let

$$\tilde{P}_t = \left(\tilde{P}_{1t}, \tilde{P}_{2t}\right) = \left(P_{1t} + \left[\hat{\Omega}^{(t)^{-1}} \hat{W}_t\right]_1 - Y_{1t}\beta_3 - \zeta_1, P_{2t} + \left[\hat{\Omega}^{(t)^{-1}} \hat{W}_t\right]_2 - Y_{2t}\beta_3 - \zeta_2\right).$$

As $\left(\tilde{P}_{1t}, \tilde{P}_{2t}\right)$ is equivalent to $(e_{1t}^c, e_{2t}^c + \lambda_{2t})$ it follows that

$$\begin{bmatrix} \tilde{P}_{1t} \\ \tilde{P}_{2t} - \lambda_{2t} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \sigma_c^2 \begin{bmatrix} R_{11}(\psi_c) & R_{12}(\psi_c) \\ R_{21}(\psi_c) & R_{22}(\psi_c) \end{bmatrix} \right),$$

or equivalently

$$\left[\lambda_{2t} | \tilde{P}_{1t}, \tilde{P}_{2t}\right] \sim N\left\{ \tilde{P}_{2t} - R_{21}(\psi_c) R_{11}^{-1}(\psi_c) \tilde{P}_{1t}, \sigma_c^2 \left(R_{22}(\psi_c) \quad - R_{21}(\psi_c) R_{11}^{-1}(\psi_c) R_{12}(\psi_c) \right) \right\} \tag{A-8}$$

making use of the properties of the conditional normal and subtracting π_{2t} from both sides of the resulting conditional distribution for $\left[\tilde{P}_{2t} - \lambda_{2t} | \tilde{P}_{1t}\right]$. Noting that λ_{2t} is structurally greater than zero we therefore sample λ_{2t} from the following truncated normal distribution:

$$N\left\{ \tilde{P}_{2t} - R_{21}(\psi_c) R_{11}^{-1}(\psi_c) \tilde{P}_{1t}, \sigma_\zeta^2 \left(R_{22}(\psi_c) - R_{21}(\psi_c) R_{11}^{-1}(\psi_c) R_{12}(\psi_c) \right) \right\} I\{\lambda_{2t} > 0\}. \tag{A-9}$$

Step 3. Sample σ_ζ^2 and σ_c^2.

We use the conjugate $Inverse - Gamma\,(1, 1)$ priors for σ_ζ^2 and σ_c^2. The posterior distributions are also inverse-gamma.

Step 4. Sample ψ_ζ and ψ_c.

Use a discrete sampler. Assume ψ_c (or ψ_ζ) can only take n values: $(\psi_1, \ldots, \psi_n)$. For each ψ_i, calculate the posterior probability

$$|R_J(\psi_i)|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2\sigma_c^2}\sum_{t=1}^{T}(\pi_t - \lambda_t)^T R_J^{-1}(\psi_i)(\pi_t - \lambda_t)^T\right\}$$

$$\left(\text{or } |R_J(\psi_i)|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2\sigma_\zeta^2}\zeta^T R_J(\psi_i)\zeta\right\}\right).$$

Sample ψ_c (or ψ_ζ) with replacement from $(\psi_1, \ldots, \psi_n)$.

Appendix II: Contraction Mapping

We establish the existence and uniqueness of the equilibrium prices for the simulation given that $\gamma_i > 0$ and $c_j > 0$ for all i and j. Let $q \geq 0$ be a real number sufficiently large and $\bar{\gamma} = E(\gamma_i)$. We use the *constructive* approach to show that the following nonlinear vector function

$$f_j(P) = P_j + \ln(c_j + q) - \ln\left(P_j + \left[\Omega^{-1}W(P)\right]_j + q\right), \tag{A-10}$$

$$g_j(P) = P_j - \frac{1}{\bar{\gamma}}\ln\frac{K_j}{I} + \frac{1}{\bar{\gamma}}\ln W_j(P), \tag{A-11}$$

with J_1 functions of (A-10) and J_2 functions of (A-11) ($J_1 + J_2 = J$), is a contraction mapping within a compact and convex subset of the Euclidian space R^J. Here we suppress the index t because the same rule applies to all the t periods in the paper. Let $U = \left[0, \bar{P}\right]^J$ be this subset with a sufficiently large $\bar{P}$. Then by the contraction mapping theorem, there exist a unique solution of prices within this closed subset. Note the equivalence between our supply side model in equation (14) and equations (A-10) and (A-11) when $f_j(P) = P$.

We will write $W_j = W_j(P)$ without ambiguity and substitute the notation $\int f(\gamma_i, W_{ij})\, dF(\beta_{1i}, \beta_{2i}, \gamma_{3i})$ for any function $f(\gamma_i, W_{ij})$ with $E(f(\gamma_i, W_{ij}))$.

Lemma 1: *Functions $g_j(P)$, $j = 1, \ldots, J$ in (A-11) define a contraction mapping: $U \to U$.*

Proof: Because our system is bounded, we only need establish that $\frac{\partial g_j(P)}{\partial P_k} > 0$ and $\sum_{k=1}^{J}\frac{\partial g_j(P)}{\partial P_k} <$

1. Indeed,

$$\frac{\partial g_j(P)}{\partial P_k} = \frac{E[\gamma_i W_{ij} W_{ik}]}{E(\gamma_i) E(W_{ij})} > 0, \text{ for } k \neq j \tag{A-12}$$

because γ_i, W_{ij} and W_{ik} are all greater than 0. Because γ_i and W_{ij} are negatively correlated (the higher the γ_i, the greater the market share W_{i0} for the outside good), we have

$$cov(\gamma_i, W_{ij}) \leq 0 \Rightarrow E(\gamma_i) E(W_{ij}) \geq E[\gamma_i W_{ij}] \text{ and } \frac{E[\gamma_i W_{ij}]}{E(\gamma_i) E(W_{ij})} \leq 1. \tag{A-13}$$

Therefore, if $k = j$, we have

$$\frac{\partial g_j(P)}{\partial P_j} = \frac{E(\gamma_i) E(W_{ij}) - E[\gamma_i W_{ij}(1 - W_{ij})]}{E(\gamma_i) E(W_{ij})} \geq \frac{E\left[\gamma_i W_{ij}^2\right]}{E(\gamma_i) E(W_{ij})} > 0$$

$$\text{and } \sum_{k=1}^{J} \frac{\partial g_j(P)}{\partial P_k} = 1 - \frac{E[\gamma_i W_{ij} W_{i0}]}{E(\gamma_i) E(W_{ij})} < 1, \tag{A-14}$$

since $W_{i0} < 1$. This completes the proof. □

Lemma 2: *If all the parameters in W_{ij} are finite and $c_j \geq 0$ for all j, there exist a real number $q \geq 0$ such that functions $f_j(P)$ in (A-10) for all $j = 1, \ldots, J$ define a contraction mapping: $U \rightarrow U$.*

Proof: We shall first show this is true for the simple case where there are only homogeneous parameters in W_{ij} and single ownership of outlets. In this case, $\left[\Omega^{-1} W(P)\right]_j = -\frac{1}{\gamma} \cdot \frac{1}{1-W_j}$. We need select a number $q \geq 0$ sufficiently large such that

$$\frac{\partial f_j(P)}{\partial P_j} = 1 - \frac{1/(1 - W_j)}{P_j - \frac{1}{\gamma}\frac{1}{1-W_j} + q} > 0, \text{ if } k = j$$

$$\frac{\partial f_j(P)}{\partial P_k} = \frac{W_j W_k / (1 - W_j)^2}{P_{tj} - \frac{1}{\gamma}\frac{1}{1-W_j} + q} > 0, \text{ if } k \neq j,$$

$$\sum_{k=1}^{J} \frac{\partial f_j(p_t)}{\partial p_{tk}} = 1 - \frac{1 - W_0 W_j}{P_{tj} - \frac{1}{\gamma}\frac{1}{1-W_j} + q} < 1.$$

Note that none of W_j, $j = 1, \ldots, J$ can approach 0 or 1 infinitesimally because P_j can at most be $\bar{P}$ and W_0 is greater than 0 for the outside good even when all $P_j = 0$. Because all three numerators in the expressions above are bounded, we can select a real number q sufficiently large such that all the conditions are satisfied.

For the more complicated case when the parameters in W_{ij} are modeled heterogeneously and firms own multiple outlets, we can apply the same method to find a q large enough because all W_{ij} and $1-W_{ij}$ are bounded. This completes the proof. $\square$

Selecting a right q can be a challenge in real practice. However, for the Roanoke apartment data in this paper, we find that $q=0$ works perfectly because $P_j + \left[\Omega^{-1} W(P)\right]_j$ is in the neighborhood of $c_j \approx 240$, which is itself large enough to satisfy the conditions for the contraction mapping .

Lemma 3: *Let $x \in R^M$ and $y \in R^N$. Let $f(x,y)$ and $g(x,y)$ be two contraction mappings: $R^{M+N} \to R^{M+N}$. Let $f(x,y) = \left(f^{(1)}(x,y), f^{(2)}(x,y)\right)$ and $g(x,y) = \left(g^{(1)}(x,y), g^{(2)}(x,y)\right)$ where $f^{(1)}(x,y)$ and $g^{(1)}(x,y)$ are mappings: $R^M \to R^M$ and $f^{(2)}(x,y)$ and $g^{(2)}(x,y)$ are mappings: $R^N \to R^N$. Let $h(x,y) = \left(f^{(1)}(x,y), g^{(2)}(x,y)\right)$, then $h(x,y)$ is a contraction mapping: $R^{M+N} \to R^{M+N}$.*

Proof: Let x_j be the jth coordinates of vector x. Using the sup-norm metric

$$d\left\{x', x\right\} = \sup\{\left|x'_1 - x_1\right|, \ldots, \left|x'_M - x_M\right|\},$$

and by the definition of contraction mappings, we have

$$d\left\{f\left(x',y'\right), f(x,y)\right\} \leq K_1 d\left\{\left(x',y'\right),(x,y)\right\} \text{ and } d\left\{g\left(x',y'\right), g(x,y)\right\} \leq K_2 d\left\{\left(x',y'\right),(x,y)\right\},$$

where $0 \leq K_1,\ K_2 < 1$.

$$\begin{aligned}
d\left\{h\left(x',y'\right), h(x,y)\right\} &= \sup\left\{\begin{array}{c} \left|f_1^{(1)}(x',y') - f_1^{(1)}(x,y)\right|, \ldots, \left|f_M^{(1)}(x',y') - f_M^{(1)}(x,y)\right|, \\ \left|g_1^{(2)}(x',y') - g_1^{(2)}(x,y)\right|, \ldots, \left|g_N^{(2)}(x',y') - g_N^{(2)}(x,y)\right| \end{array}\right\} \\
&\leq \sup\left\{\begin{array}{c} \sup\left\{\begin{array}{c} \left|f_1^{(1)}(x',y') - f_1^{(1)}(x,y)\right|, \ldots, \left|f_M^{(1)}(x',y') - f_M^{(1)}(x,y)\right|, \\ \left|f_1^{(2)}(x',y') - f_1^{(2)}(x,y)\right|, \ldots, \left|f_N^{(2)}(x',y') - f_N^{(2)}(x,y)\right| \end{array}\right\}, \\ \sup\left\{\begin{array}{c} \left|g_1^{(1)}(x',y') - g_1^{(1)}(x,y)\right|, \ldots, \left|g_M^{(1)}(x',y') - g_M^{(1)}(x,y)\right|, \\ \left|g_1^{(2)}(x',y') - g_1^{(2)}(x,y)\right|, \ldots, \left|g_N^{(2)}(x',y') - g_N^{(2)}(x,y)\right| \end{array}\right\} \end{array}\right\} \\
&= \sup\left\{d\left\{f\left(x',y'\right), f(x,y)\right\}, d\left\{g\left(x',y'\right), g(x,y)\right\}\right\} \\
&\leq \sup\left\{K_1, K_2\right\} d\left\{\left(x',y'\right),(x,y)\right\},
\end{aligned}$$

and $0 \leq \sup\{K_1, K_2\} < 1$. Therefore $h(x, y)$ is a contraction mapping. $\square$

Theorem: *The price simulation that converges eventually to a mapping: $U \to U$ with J_1 functions of (A-10) and J_2 functions of (A-11) has a unique solution.*

Proof: Lemma 1 and 2 show that $J = J_1 + J_2$ functions of either (A-10) or (A-11) are contraction mappings. Lemma 3 shows that the mixed system on $U \to U$ with J_1 functions of (A-10) and J_2 functions of (A-11) is also a contraction mapping. Because $U = \left[0, \bar{P}\right]^J$ is compact and convex, there is a unique fixed point $P \in U$ such that $P = (f(P), g(P))$. $\square$

Appendix III: Simulation Design

To approximate the size and structure of the data we randomly distribute 40 apartment locations in a rectangular area shown in Figure 7. For purposes of parsimony, the single complex-specific attribute considered is the presence of a garage. The single simulated location attribute is the distance to the town center, marked by a diamond in Figure 7. The utility function is (1), where X_{tj} is a 0-1 indicator variable that represents the presence of the garage and X_{ts_j} is the distance to the town center. X_{tj} and X_{ts_j} do not vary over time. The diagonal distance across this rectangular region is 30 units. We set $\beta_1 = 1.0$, $\beta_2 = -0.3$, and $\gamma = 0.01$. The spatial random effect θ_{ts_j} is distributed $N\left(0, \sigma_\theta^2 R_J(\phi)\right)$. Using the exponential covariance function for the Gaussian process, we set $\sigma_\theta = 0.4$ and $\phi_\theta = 0.4$. We simulate data for 10 independent periods with 10 independently sampled θ_t's.

[Insert Figure 7 Here]

The prices for the 40 apartments and 10 periods are determined by the full model. The cost for each apartment is simulated using (7), where Y_{tj} includes an intercept and a scalar ($Y_{tj} = (1, Y_{tj,1})$ and $\beta_3 = \left(\beta_{31,}\beta_{32}\right)$) representing the size of the apartment complex's rooms (we presume larger rooms cost more to maintain). We assume three values: $(1, 2, 3)$ for $Y_{tj,1}$. We set $\beta_{31} = 150$ and $\beta_{32} = 30$. ζ_{tj} is distributed $N\left(0, \sigma_\zeta^2 R_J(\varphi)\right)$, where $\sigma_\zeta = 10$, $\varphi = 0.4$. The capacity constraint K_j for all the apartment is set to 200.

The price P_{tj} and demand Q_{tj} are generated by our demand-supply model. An iterative procedure

$$P_t^{(m+1)}(\gamma) = P_t^{(m)}(\gamma) + \ln c_t - \ln\left(P_t^{(m)} + \left[\Omega^{-1} W\left(P^{(m)}\right)\right]_j\right) \tag{A-15}$$

generates all the P_{tj}'s in each period. We then impute demand, $Q_{tj}(P_t)$. If Q_{tj} exceeds K_j, the price for the apartment, (A-15), becomes

$$P_t^{(m+1)}(\gamma) = P_t^{(m)}(\gamma) - \frac{1}{\gamma}\ln\frac{K_j}{I} + \frac{1}{\gamma}\ln W_t\left(P_t^{(m)}\right) \tag{A-16}$$

All the prices P_{tj}'s are generated again and Q_{tj}'s are computed. If any Q_{tj} exceed K_j, we replace the mapping in (A-15) by (A-16), and iterate demand and prices again. We continue in this fashion until all the equilibrium prices and quantities satisfy the capacity constraints. Using these contraction mappings, prices in our simulation converge to a equilibrium that is unique over a wide range of starting values, as proved in Appendix II.

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Table 1.– Test of Instrumental Variables

Period	J-statistic	p-value(χ^2_7 under null hypothesis)	Shea's R^2
2	7.38	0.39	0.25
3	6.86	0.44	0.26
4	6.93	0.44	0.33
5	7.25	0.40	0.30
6	8.02	0.33	0.31
7	8.31	0.31	0.29

Table 2 – Data Descriptives

Attribute	**Observations**	**Mean**	**Std. Dev.**	**Max**	**Min**
Price (Dollars)	364	491	102	753	290
Capacity (Units)	364	163	79	636	63
Number of Vacancies	364	10	17	150	0
Clubhouse	52	35%	48%	1	0
Tennis	52	32%	47%	1	0
Swimming Pool	52	77%	42%	1	0
Gymnasium	52	20%	41%	1	0
Heat Included	52	30%	46%	1	0
Year Built	52	1976	11	2002	1952
Average Distance to Major Employers (Miles)	52	3.91	0.86	5.7	2.28
Distance to Nearest School (Miles)	52	1.34	0.74	3.05	0.10
Maximum Distance Between Apartments (Miles)	1	9.6	—	—	—

Table 3 Simulation Result

Parameter	Variable	Simulation Value	Our Model	No Spatial Correlation	No Capacity Constraints
Demand-side			Posterior Median (95% Posterior Prediction Interval)		
β_1	Garage	1.00	1.01 (0.87,1.12)	0.94 (0.85,1.03)	—
β_2	Distance to City	-0.30	-0.29 (-0.37,-0.22)	-0.28 (-0.36,-0.19)	—
γ	Price	0.01	0.0096 (0.0078,0.012)	0.0077 (0.0051,0.0089)	—
σ_γ	Std. Dev. γ	0.002	0.0017 (0.0012,0.0026)	0.0018 (0.0013,0.0026)	—
σ_θ	Std. Dev. θ	0.40	0.38 (0.33,0.46)	0.42 (0.32,0.51)	—
ϕ_θ	Spatial Decay θ	0.40	0.56 (0.20,1.02)	—	—
σ_ν	Std. Dev. ν	0.30	0.29 (0.25,0.32)	0.35 (0.30,0.42)	—
ϕ_ν	Spatial Decay ν	0.40	0.44 (0.20,0.56)	—	—
	Log Marginal Likelihood		-27.4	-43.9	—
Supply-side			Posterior Median (95% Posterior Prediction Interval)		
$\beta_{3,0}$	Variable Cost	150	147 (141,154)	117 (103,130)	173 (166,180)
$\beta_{3,1}$	Apartment Size	30	29.6 (29.0,30.5)	29.5 (28.9,30.6)	22.2 (19.8,25.5)
σ_ς	Std. Dev. ζ	10	10.5 (9.6,11.3)	10.5 (9.6,11.2)	20.6 (17.2,25.7)
ψ_ς	Spatial Decay ζ	0.40	0.43 (0.20,0.74)	—	1.31 (0.86,1.90)
σ_c	Std. Dev. e_c	10	10.1 (9.1,11.3)	11.3 (9.9, 13.8)	17.3 (14.6,20.1)
ψ_c	Spatial Decay	0.40	0.56 (0.20,0.87)	—	1.00 (0.74,1.47)
$\lambda^{\max}$	Kuhn-Tucker	113	112 (105,121)	115 (107,129)	—
	Log Marginal Likelihood		-1093	-1130	-1276

Table 4 - Roanoke Data Results

Parameter	Variable	Full Model	No Spatial Correlation	No Capacity Constraints
Demand-side		Posterior Median (95% Posterior Prediction Interval)		
$\beta_{1,1}$	Clubhouse	0.31 (0.06,0.57)	0.30 (0.03,0.58)	—
$\beta_{1,2}$	Tennis	0.55 (0.31,0.80)	0.58 (0.35,0.83)	—
$\beta_{1,3}$	Pool	0.51 (0.20,0.78)	0.45 (0.14,0.76)	—
$\beta_{1,4}$	Gym	0.15 (-0.19,0.50)	-0.02 (0.34,0.27)	—
$\beta_{1,5}$	Heat	0.32 (0.14,0.51)	0.32 (0.15,0.50)	—
$\beta_{2,1}$	Distance to School	0.03 (-0.16,0.23)	0.05 (-0.18,0.28)	—
γ	Price	0.0035 (0.0024,0.0046)	0.0032 (0.0022,0.0042)	—
σ_γ	Std. Dev. γ	0.00057 (0.00030,0.00084)	0.00058 (0.00033,0.00085)	—
σ_θ	Std. Dev. θ	0.60 (0.50,0.72)	0.72 (0.60,0.84)	—
ϕ_θ	Spatial Decay θ	0.84 (0.39,1.38)	—	—
σ_ν	Std. Dev. ν	0.11 (0.10,0.12)	0.11 (0.10,0.13)	—
ϕ_ν	Spatial Decay ν	2.8 (2.0,3.73)	—	—
Log Marginal Likelihood		-2.84	-20.9	—
Supply-side		Posterior Median (95% Posterior Prediction Interval)		
$\beta_{3,0}$	Variable cost intercept	432 (368,498)	408 (346,472)	405 (340,465)
$\beta_{3,1}$	Heat	70.2 (33.5,107.8)	61.6 (25.2,98.6)	57.1 (22.3,90.3)
$\beta_{3,2}$	Age	-7.3 (-9.3,-5.2)	-7.2 (-9.3,-5.3)	-6.7 (-8.7,-4.6)
σ_ς	Std. Dev. ζ	88.2 (75.1,106.0)	91.0 (78.7,108.3)	102.0 (90.3,117.8)
ψ_ζ	Spatial Decay	2.8 (0.46,5.66)	—	1.9 (1.0,5.6)
σ_c	Std. Dev. e_c	26.6 (24.8,28.8)	27.0 (25.4,29.2)	34.7 (30.2,39.7)
ψ_c	Spatial Decay	3.2 (0.46,6.00)	—	4.2 (1.6,7.5)
$\lambda^{\max}$	Kuhn-Tucker	88.5 (48.2,129.6)	65.2 (26.1,118.4)	—
Log Marginal Likelihood		-1362	-1384	-1503

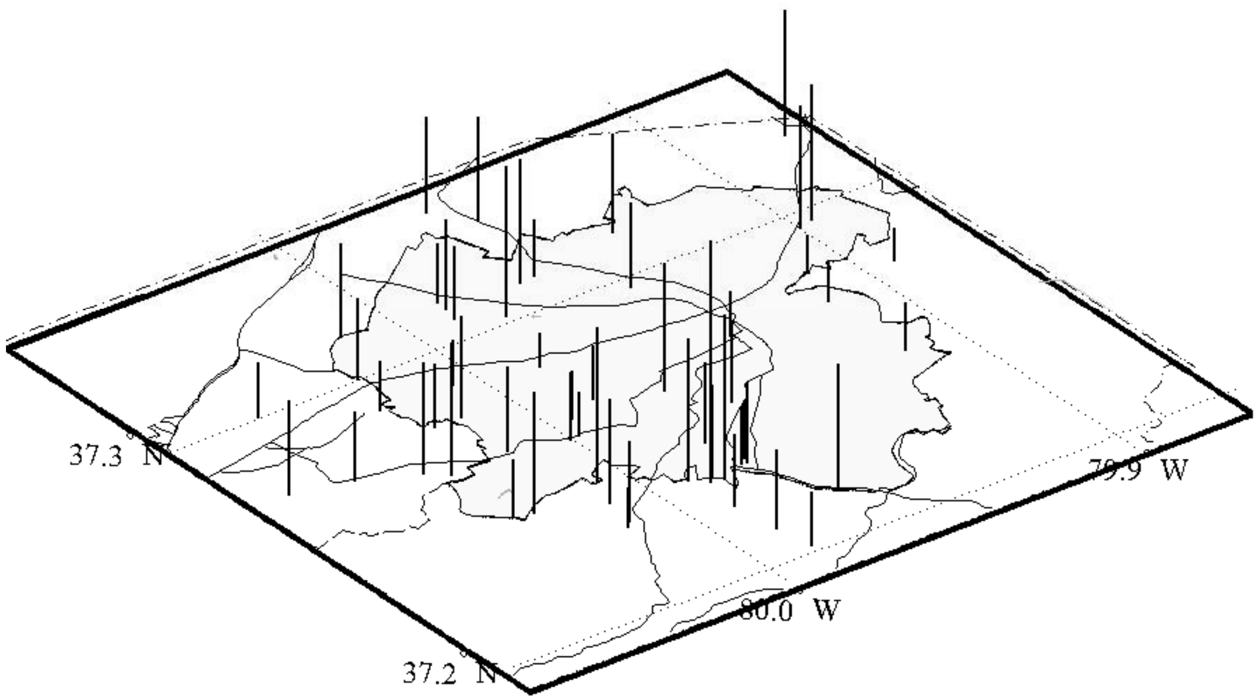


Figure 1: 2005 Location and Occupancy of Apartments in Roanoke, Virginia

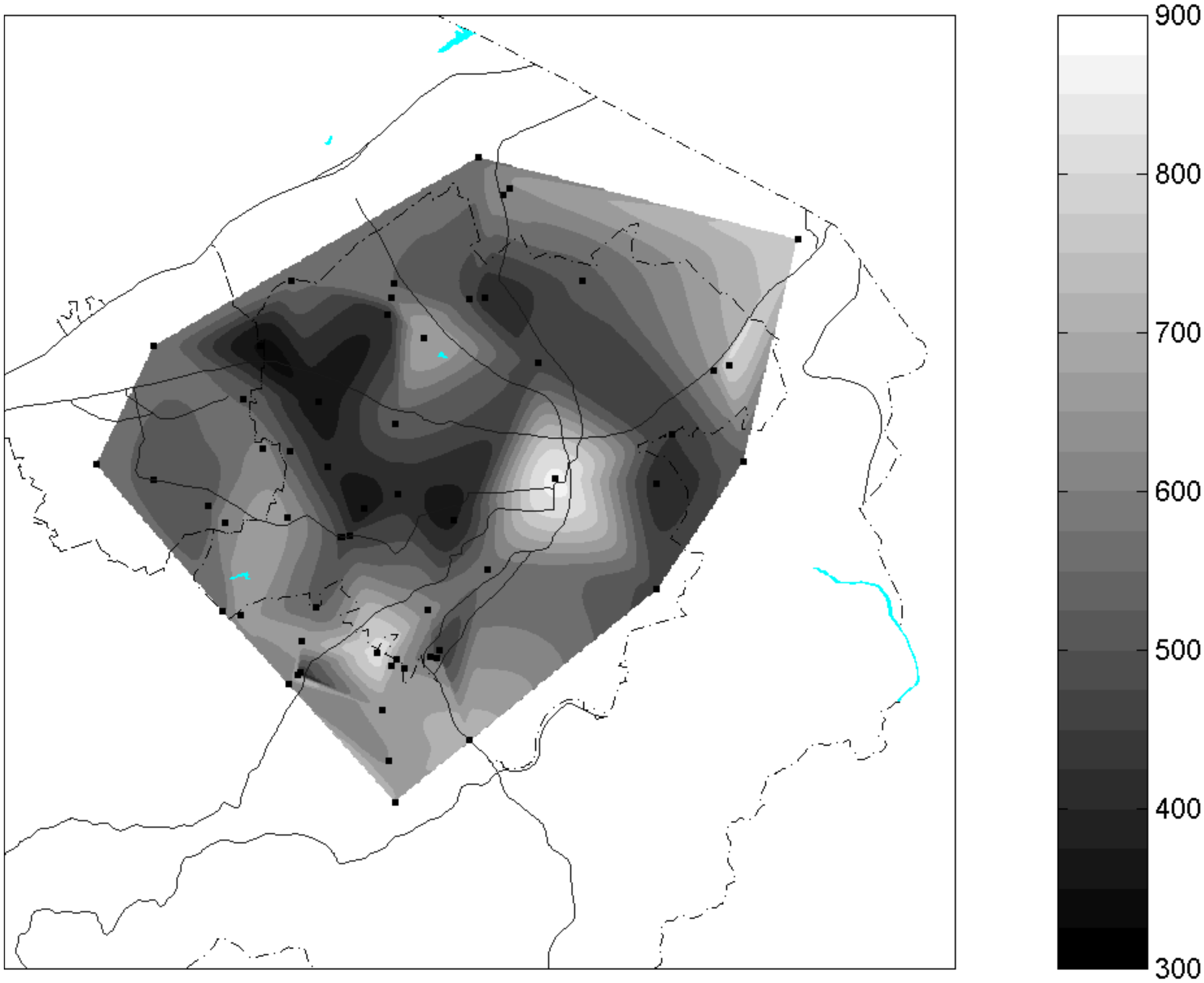


Figure 2: Distribution of 2005 Rental Prices ($) in Roanoke, Virginia

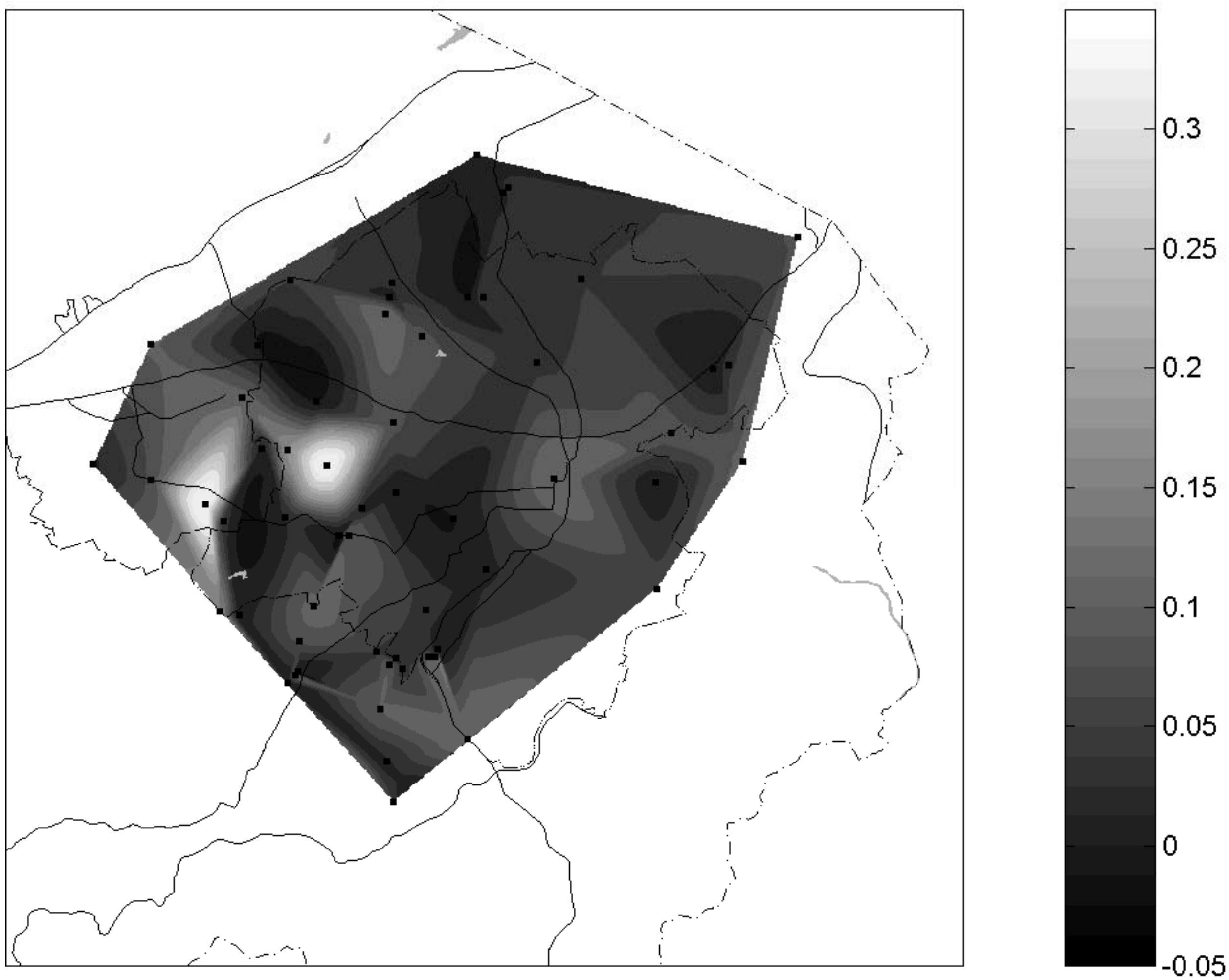


Figure 3: Distribution of 2005 Vacancy Rates (%) in Roanoke, Virginia

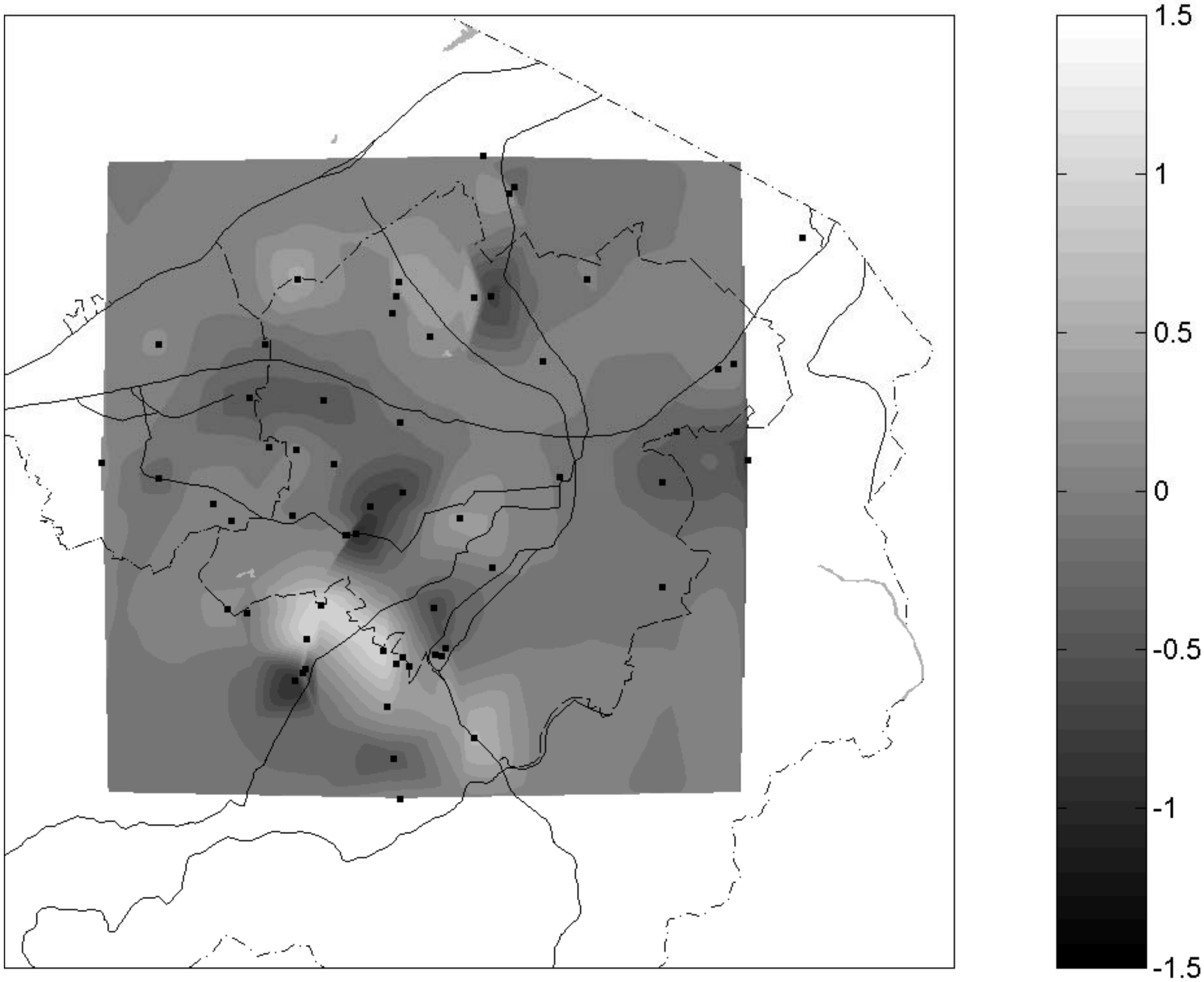
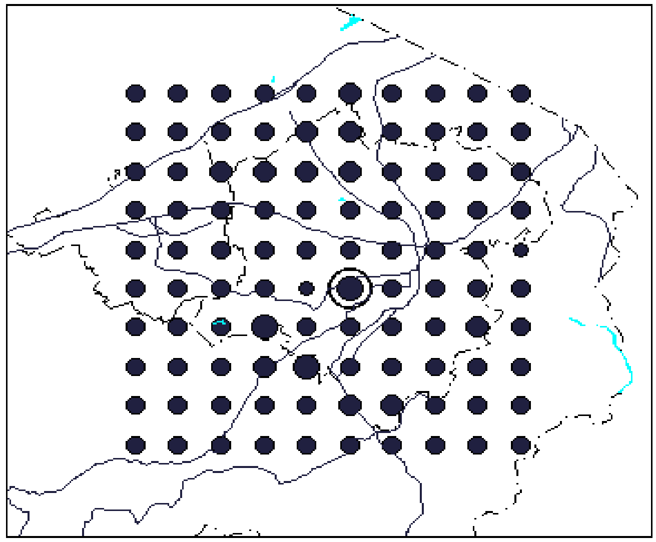


Figure 4: Latent Spatial Demand for Apartments in Roanoke, Virginia

Kriged Profits in the Full Model



Kriged Profits Omitting Spatial Effects and Capacity

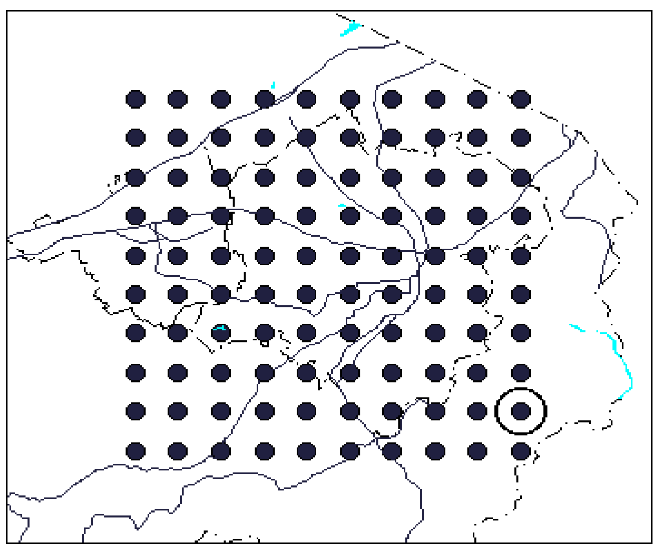


Figure 5: Kriged Profits for Potential Apartment Locations in Roanoke Virginia

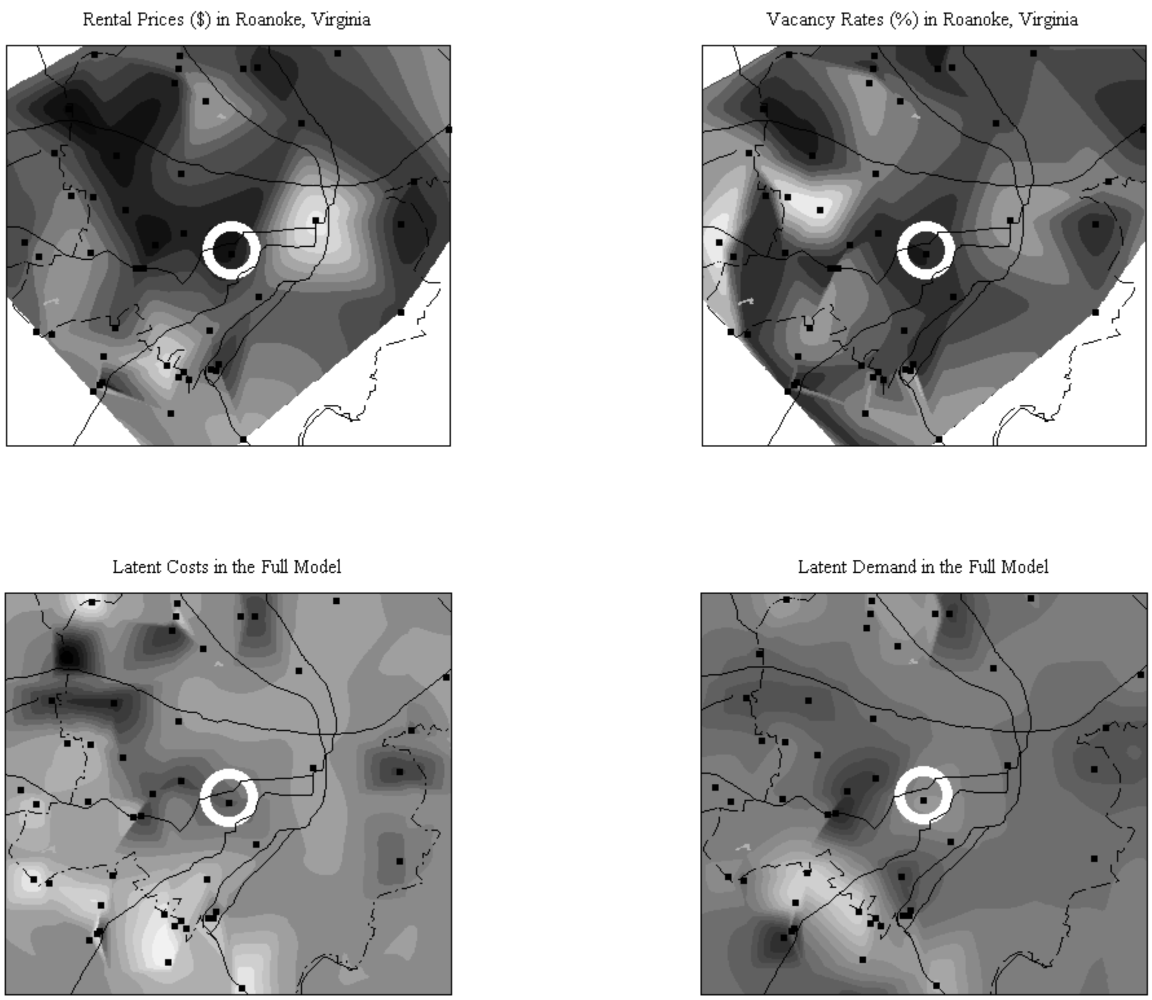


Figure 6: Optimal Location (White Circle) Superimposed on Rent, Vacancy, Cost and Demand

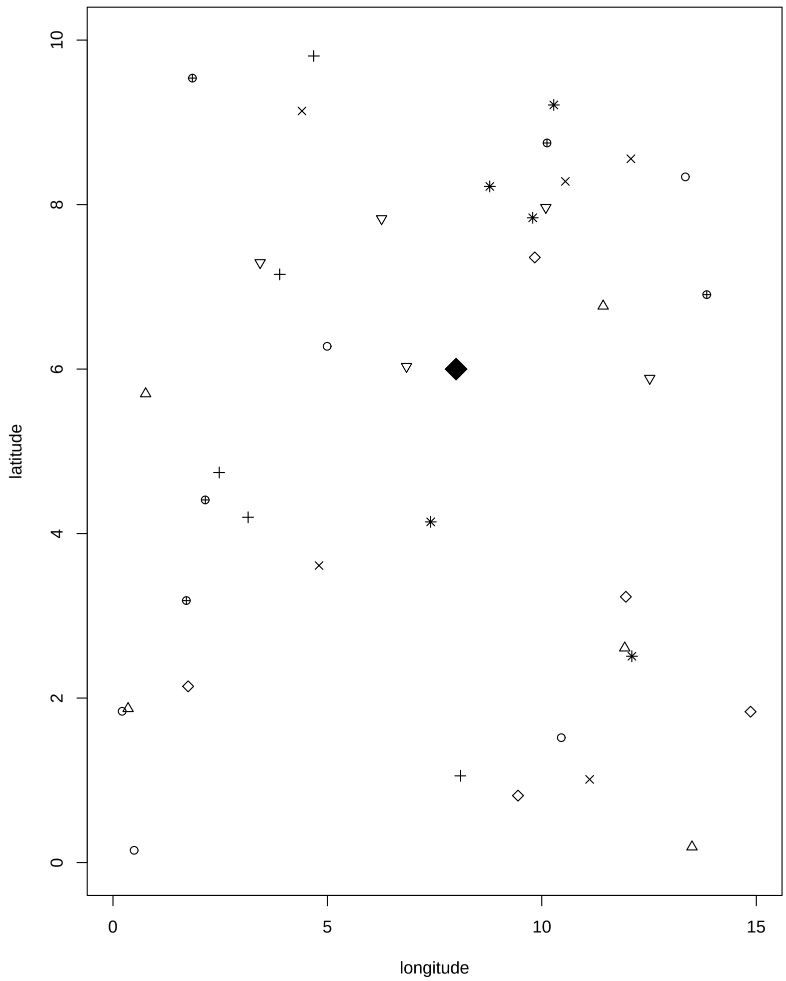


Figure 7: Chain locations (apartments from the same chain are indicated by the same symbol).