
MARKOV DECISION MODELS: AN INTRODUCTION

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OUTLINE

BASIC CONCEPTS

SIMULATION

APPLICATIONS

STROKE

BREAST CANCER

EPIDEMIOLOGY, CLAIMS

RANDOMIZED CLINICAL TRIALS

EXPERT INPUT



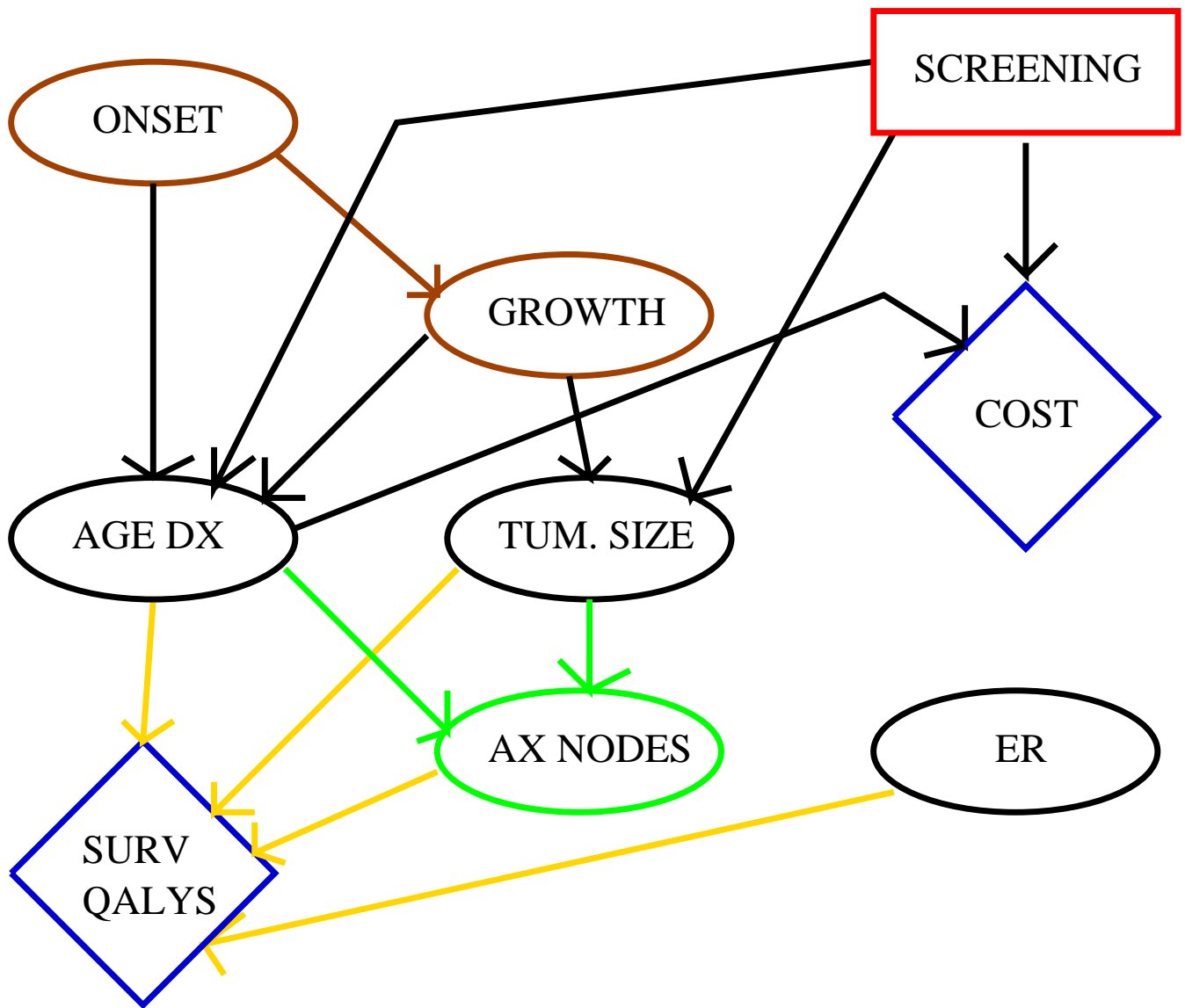
COVARIATES

UTILITIES

INTERVENTIONS

HEALTH OUTCOMES

COSTS

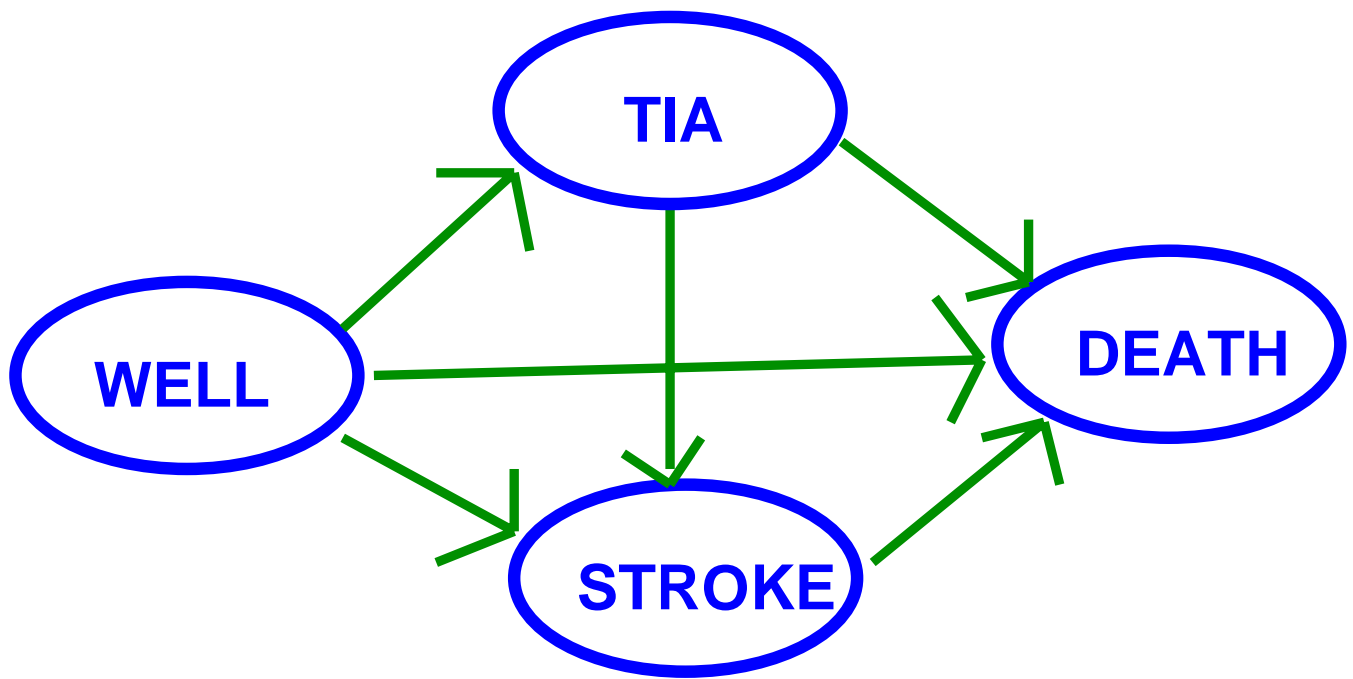


TIA

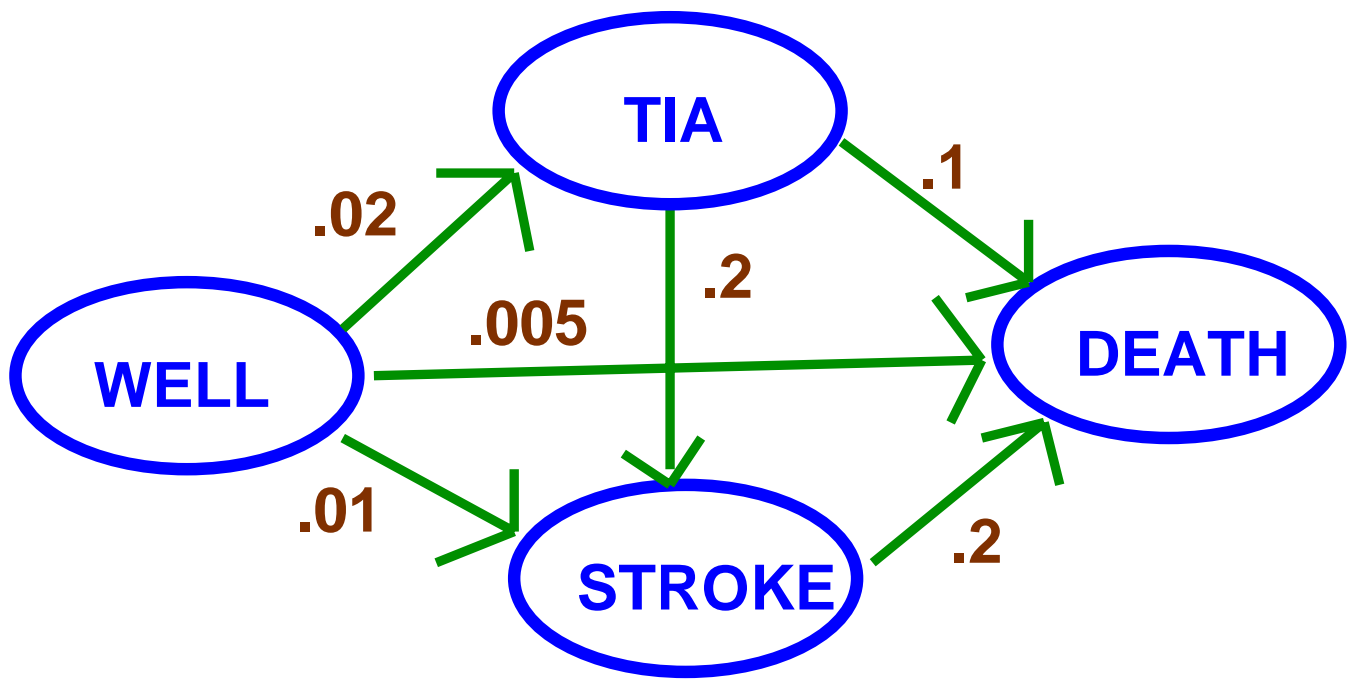
WELL

DEATH

STROKE



STATES
TRANSITIONS



STATES

TRANSITIONS

TRANSITION PROBABILITIES

MARKOV PROPERTY

PROCESS:

description of population of patient histories

MARKOV PROCESS:

transition probabilities

depend only on the current state

STATIONARY MARKOV PROCESS

transition probabilities don't change over time

LIFE EXPECTANCY

CYCLE	WELL	TIA	STROKE	DEAD
0	100000	0	0	0
1	96500	2000	1000	500
2	93122.5	3330	2165	1382.5
...				
∞	0	0	0	1
Sums	S_{well}	S_{tia}	S_{stroke}	

Expected time in well state:

$$T_{well} = S_{well} / N_{total}$$

Life expectancy:

$$T = (S_{well} + S_{tia} + S_{stroke}) / N_{total}$$

LIFE EXPECTANCY via MATRICES I

next cycle

current cycle

$$\begin{bmatrix} N_{well} \\ N_{tia} \\ N_{stroke} \\ N_{dead} \end{bmatrix} = \begin{bmatrix} 0.965 & 0.0 & 0.0 & 0 \\ 0.020 & 0.7 & 0.0 & 0 \\ 0.010 & 0.2 & 0.8 & 0 \\ 0.005 & 0.1 & 0.2 & 1 \end{bmatrix} \begin{bmatrix} N_{well} \\ N_{tia} \\ N_{stroke} \\ N_{dead} \end{bmatrix}$$

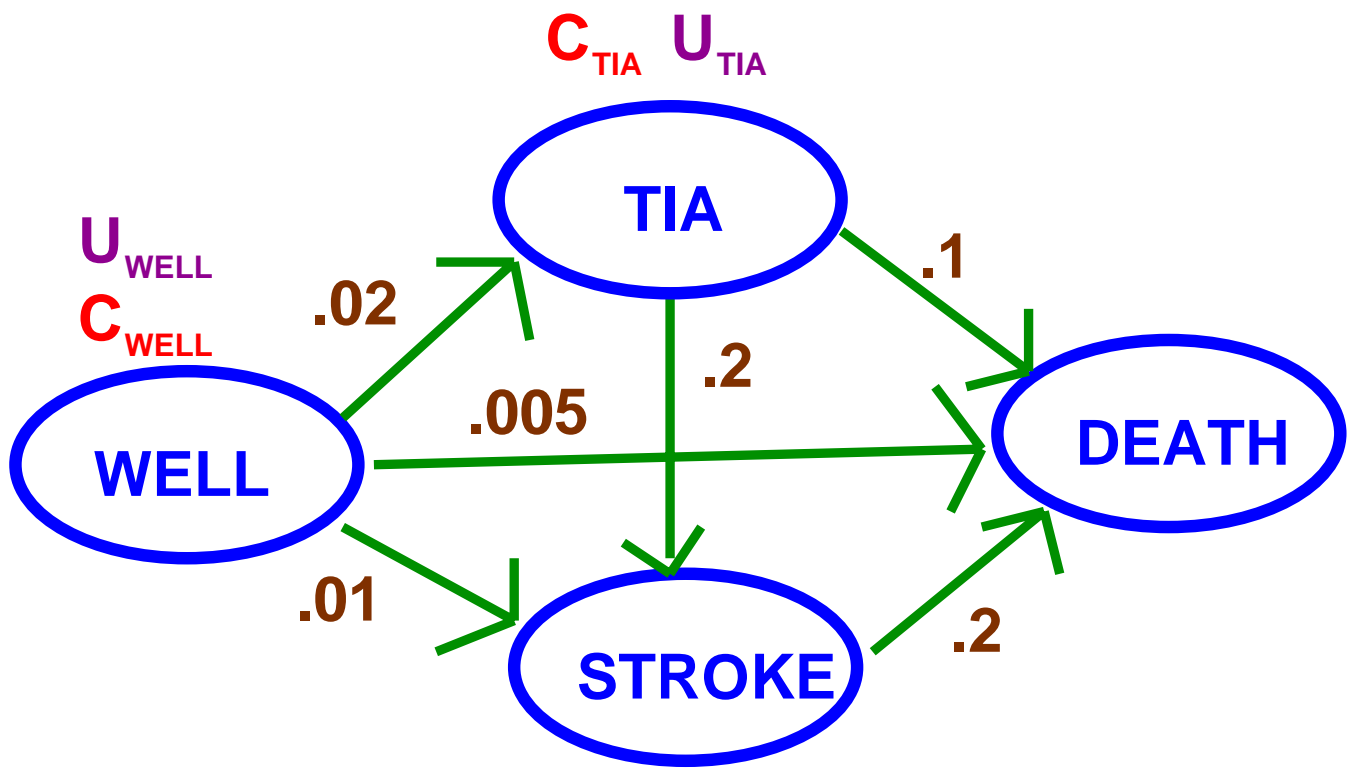
$$\mathbf{N}_{c+1} = \mathbf{P} \mathbf{N}_c$$

LIFE EXPECTANCY via MATRICES II

$$N_4 = P P P P N_0$$

$$N_c = P^c N_0$$

$$T = \text{SUM over cycles } (P^c N_0)$$



STATES

TRANSITIONS

TRANSITION PROBABILITIES

UTILITIES

COSTS

EXPECTED UTILITY AND COST

EXPECTED UTILITY (QALY):

$$U_{well} T_{well} + U_{tia} T_{tia} + U_{stroke} T_{stroke}$$

EXPECTED COST (\$):

$$C_{well} T_{well} + C_{tia} T_{tia} + C_{stroke} T_{stroke}$$

INTERVENTION EFFECTS

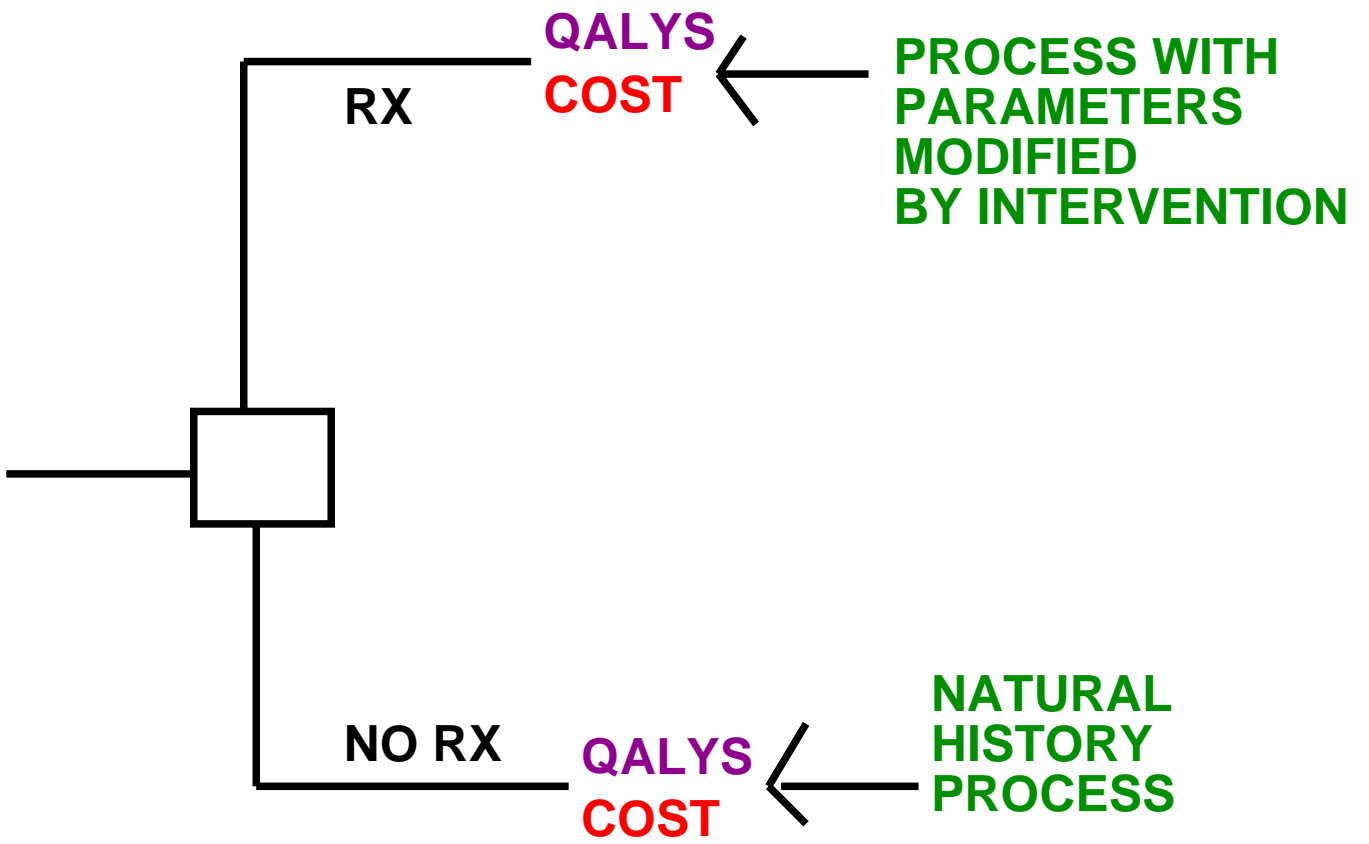
INTERVENTIONS MAY MODIFY:

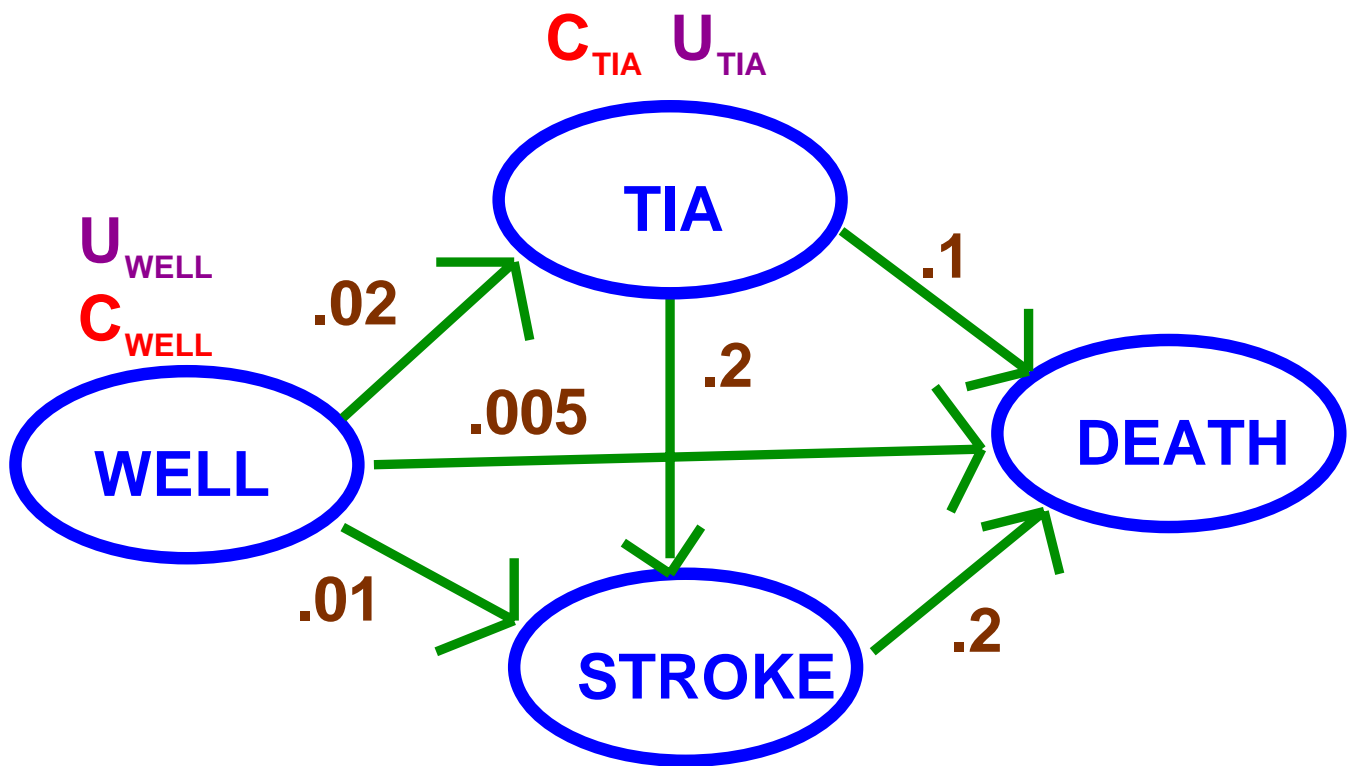
HEALTH STATES

TRANSITION PROBABILITIES

UTILITIES

COSTS





STATES

TRANSITIONS

TRANSITION PROBABILITIES

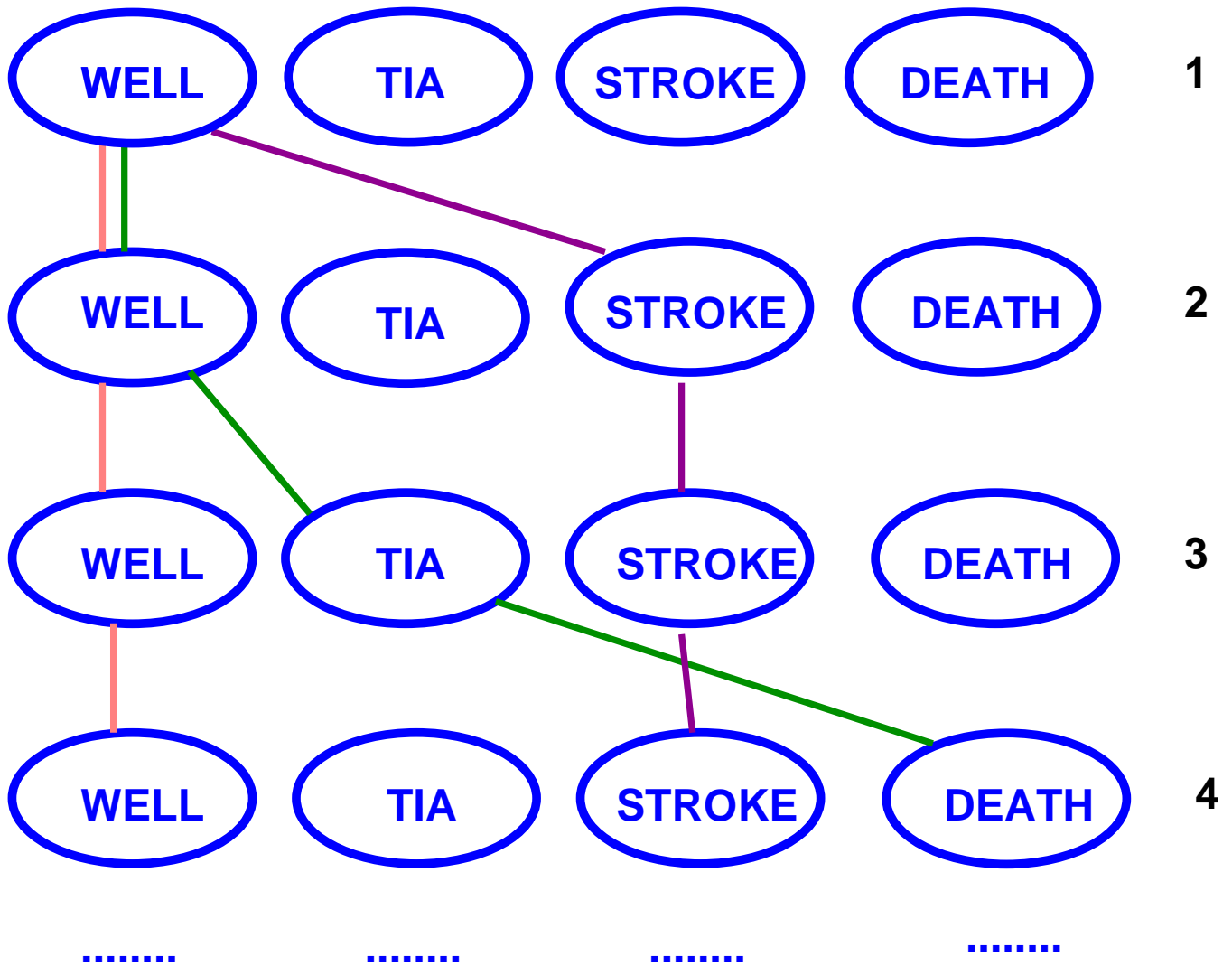
UTILITIES

COSTS

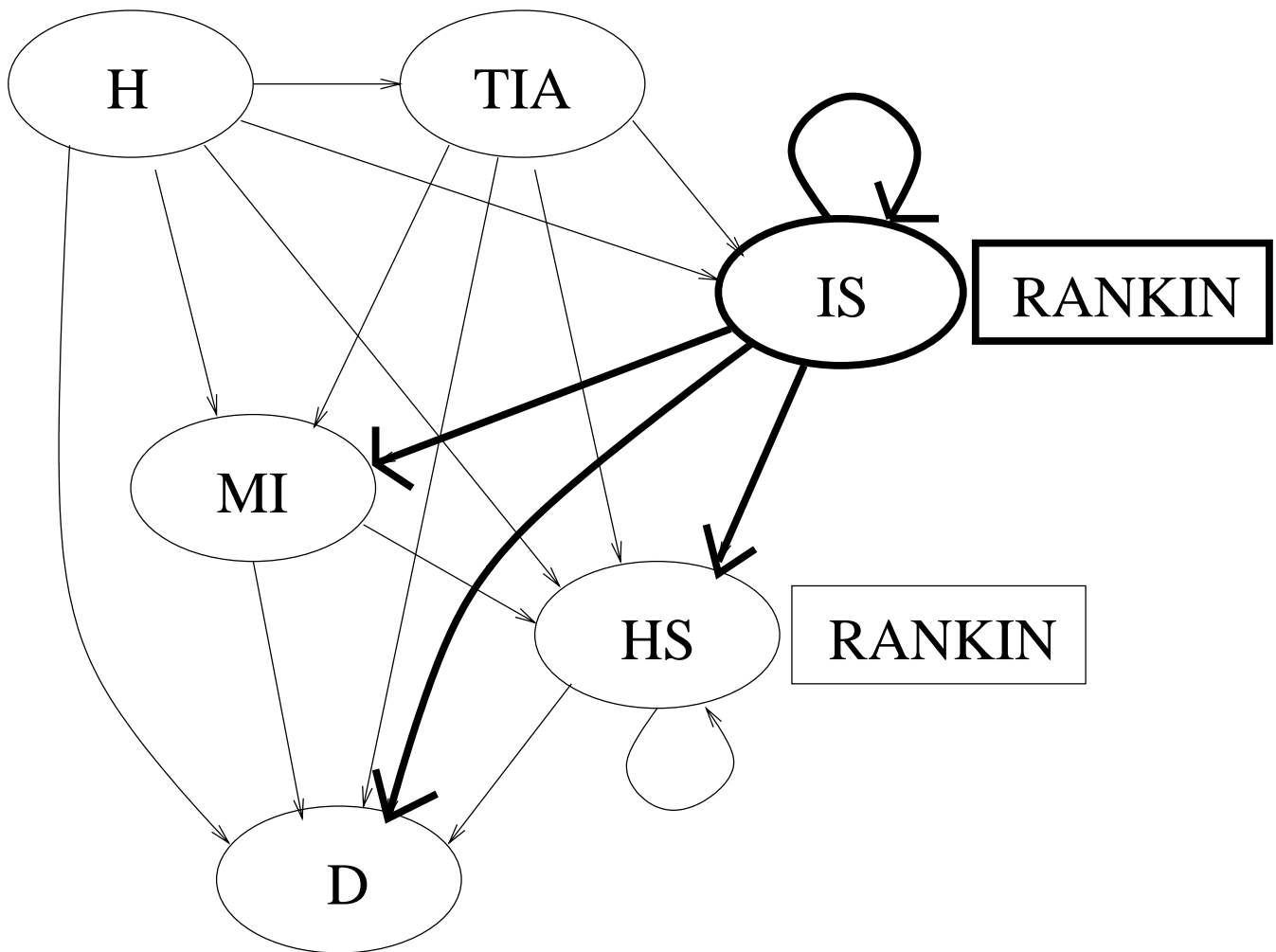
MORE SOPHISTICATED TOOLS

- **DISTRIBUTIONS, MONTE CARLO**
 - **TIME VARYING TRANSITIONS,
COSTS AND UTILITIES**
 - **UTILITIES AND COSTS FOR EVENTS**
-

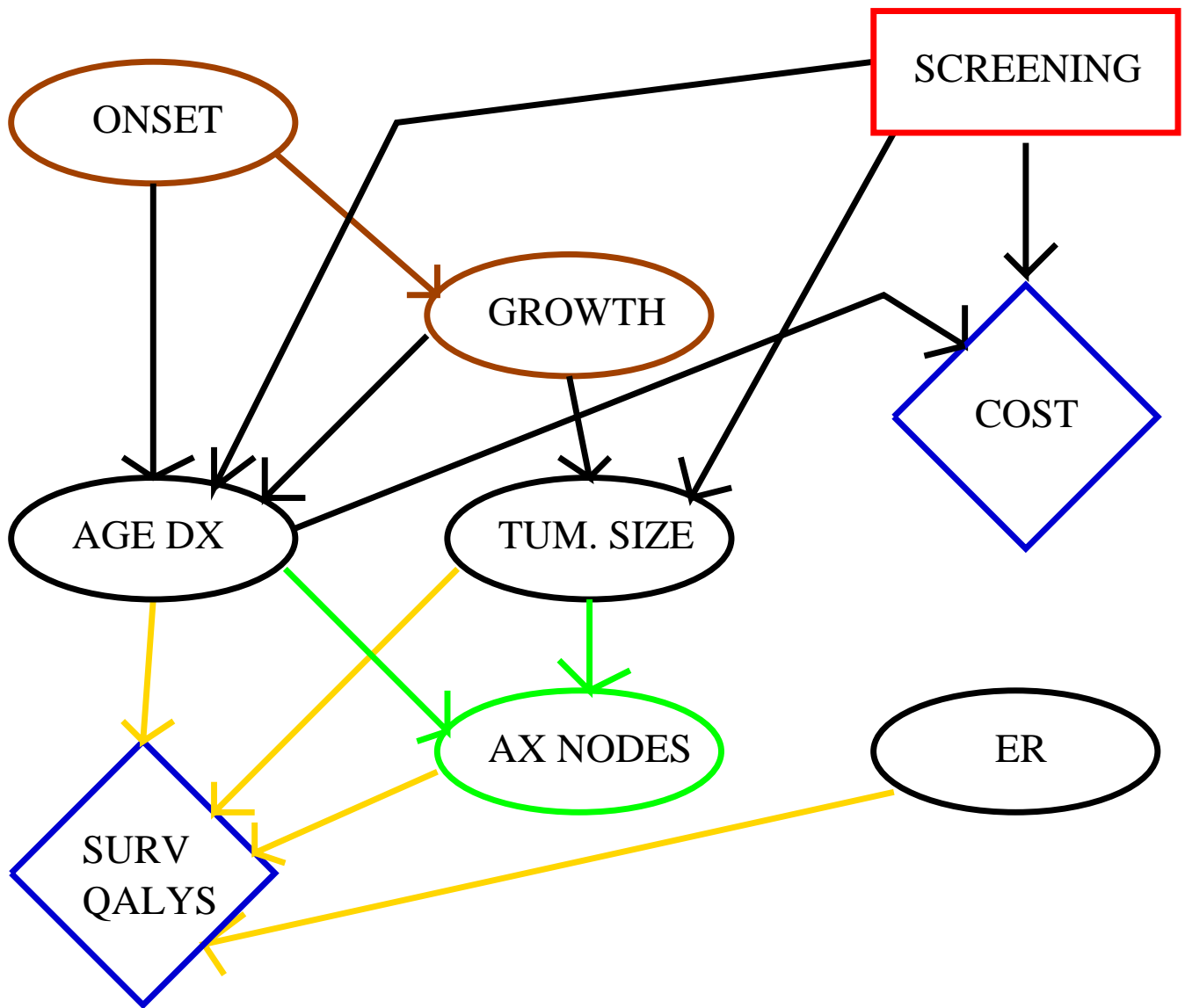
MONTE CARLO SIMULATION



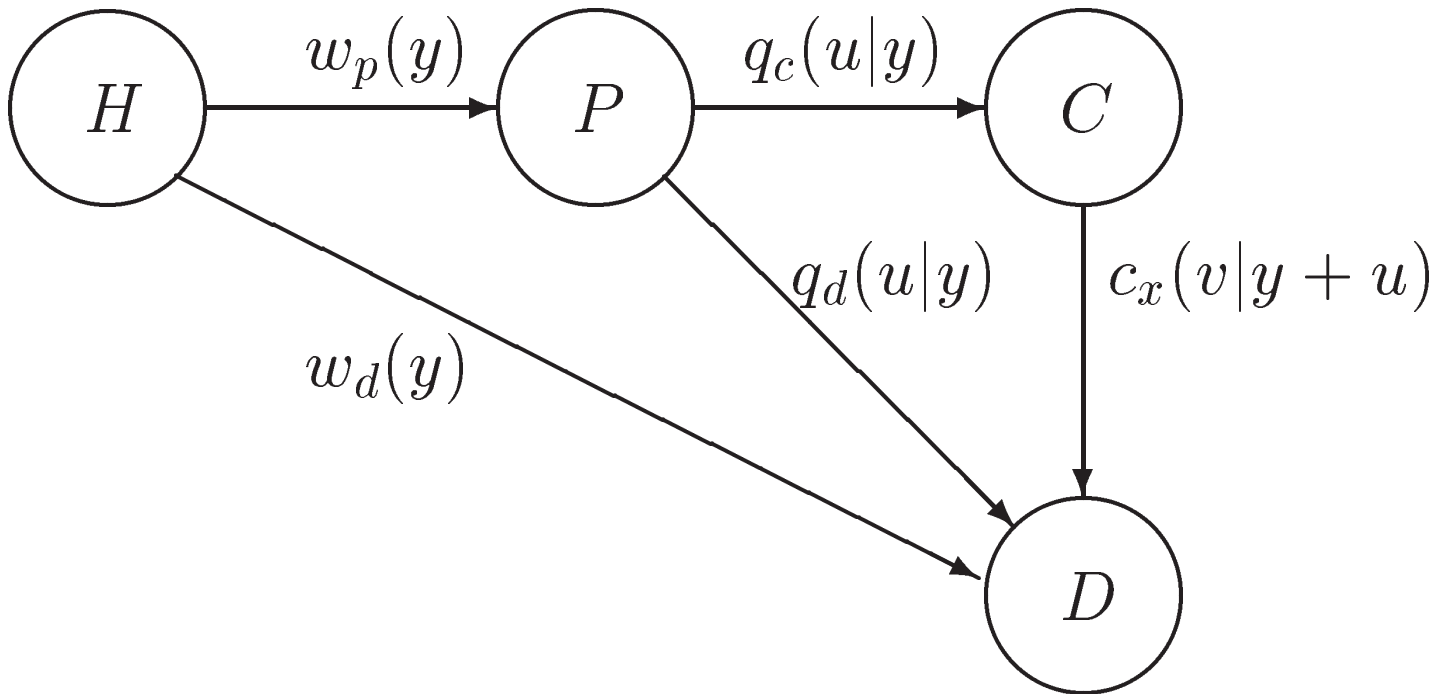
STROKE POLICY MODEL





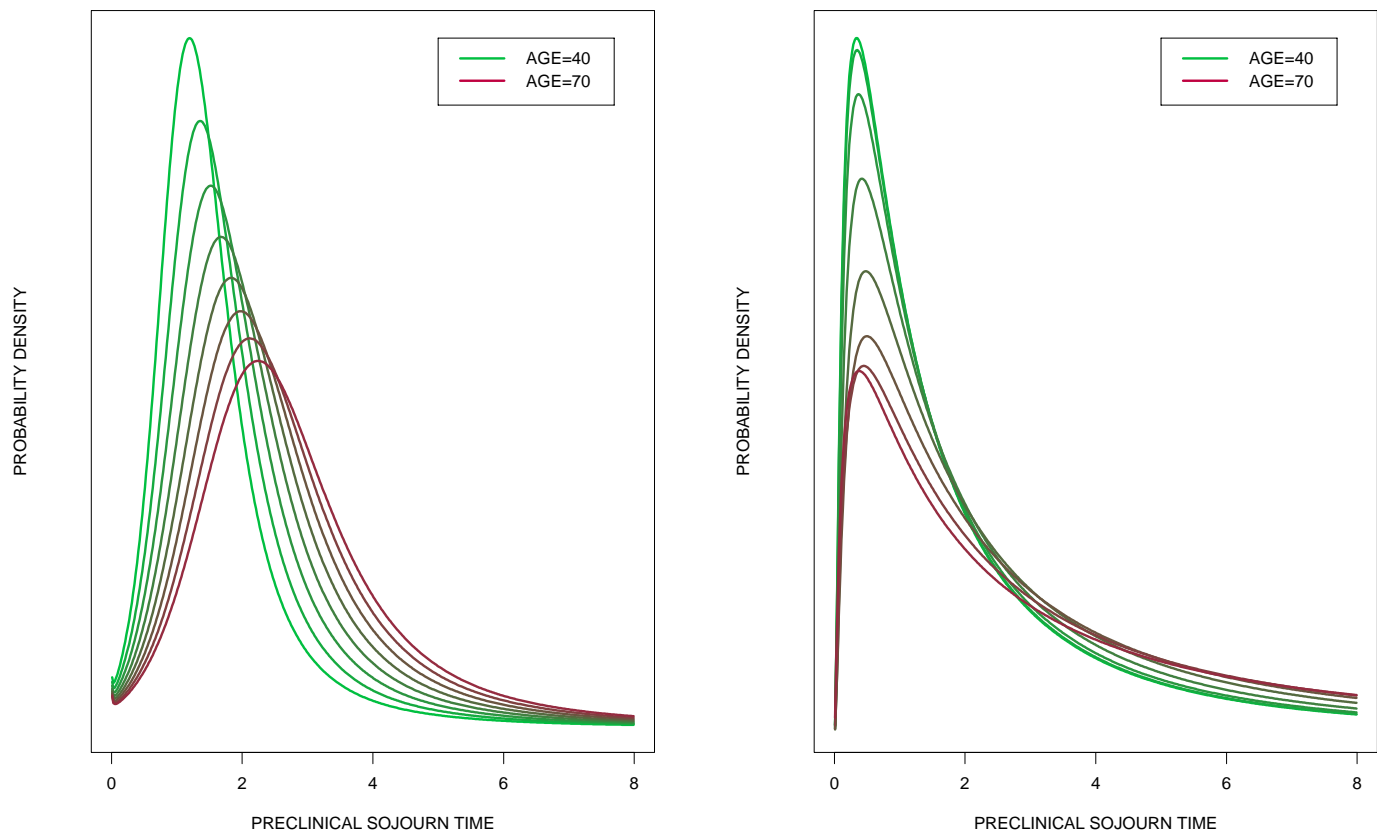


NATURAL HISTORY MODEL



Schwartz *OR*, 1978
Parmigiani *JASA*, 1993

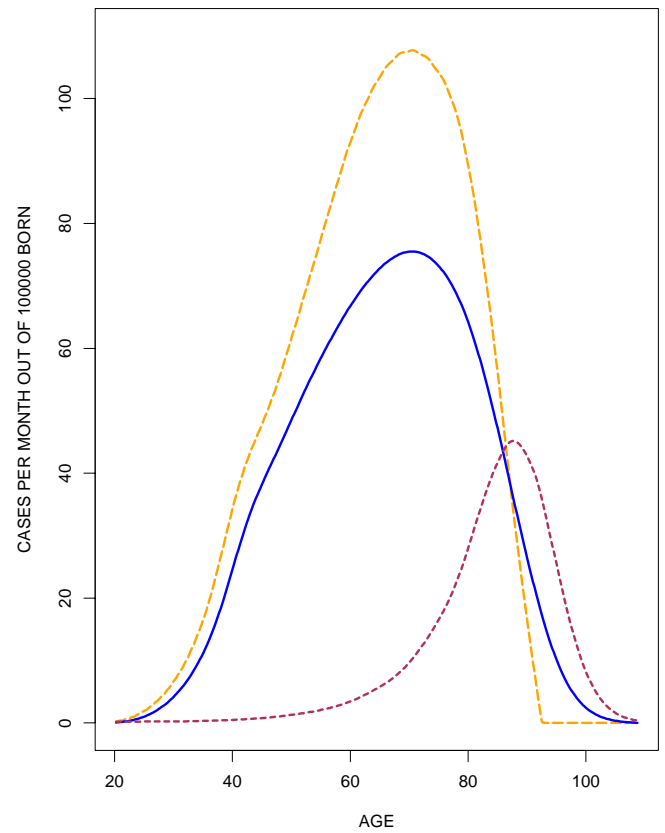
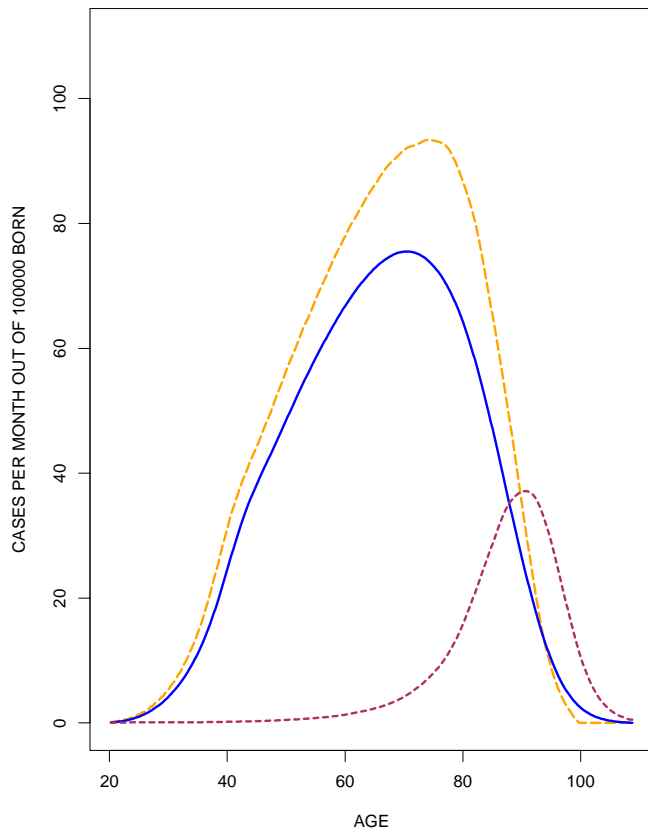
SOJOURN TIME DISTRIBUTIONS



Peer et al. *Cancer* (1993) *BCRT* (1996)

Spratt, Greenberg and Heuser *CR* (1986)

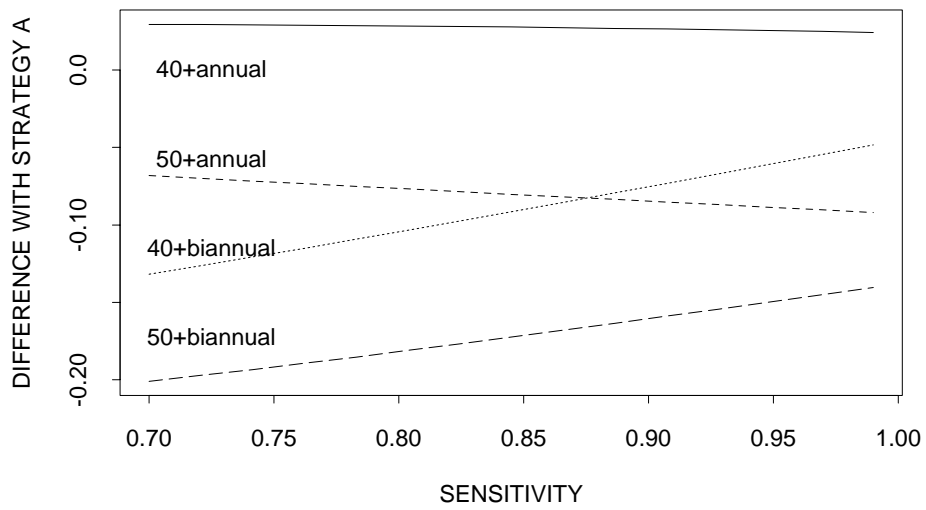
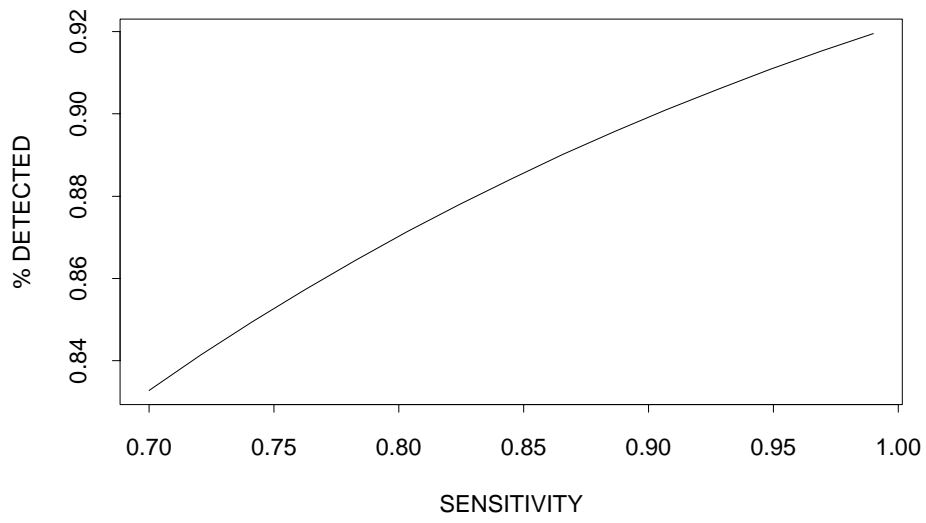
NATURAL HISTORY OF BREAST CANCER



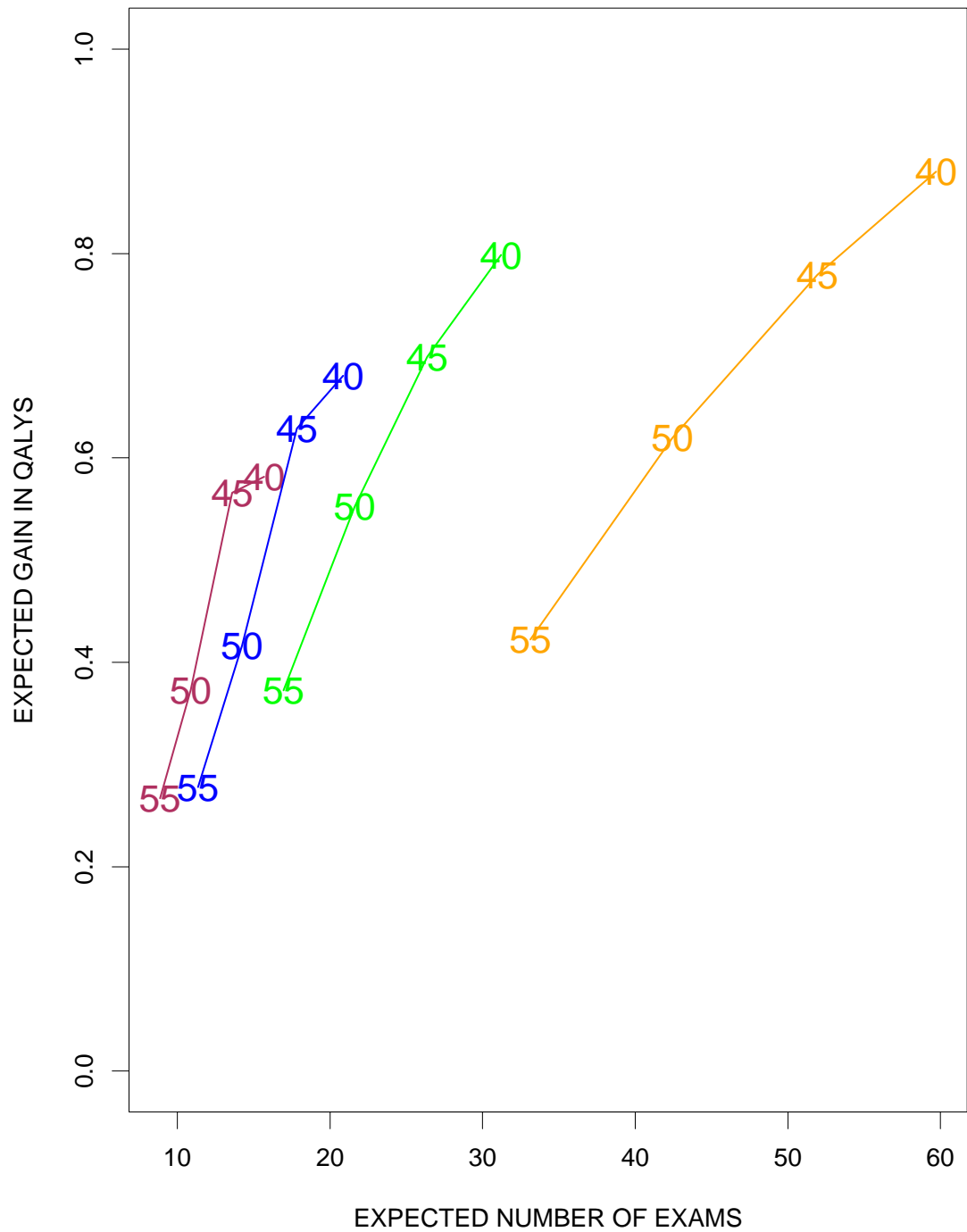
Spratt

Peer

SCREENING OUTCOMES



EXAMINATIONS-QALYS TRADEOFF I



EXAMINATIONS-QALYS II

