Computer Model Validation via Dynamic Linear Model

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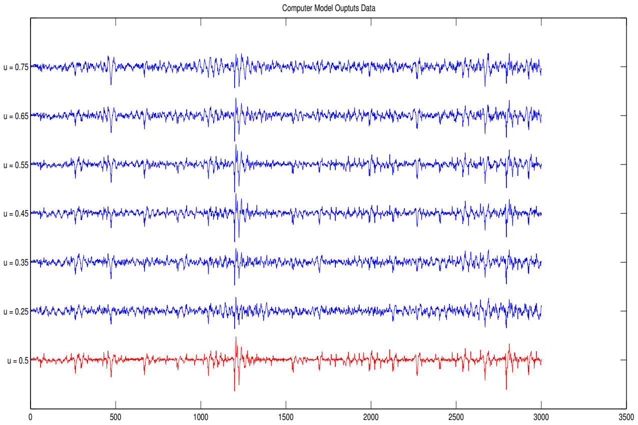
Data


Computer Model Ouptuts Data
u = 0.75
u = 0.65
u = 0.55
u = 0.45
u = 0.35
u = 0.25
u = 0.5
0
500
1000
1500
2000
2500
3000
3500

Statistical Modeling of the Spatially Correlated Computer Outputs

$$\begin{pmatrix} X_t(u_1) \\ X_t(u_2) \\ \dots \\ X_t(u_n) \end{pmatrix} = \begin{pmatrix} X_{t-1}(u_1) & X_{t-2}(u_1) & \dots & X_{t-p}(u_1) \\ X_{t-1}(u_2) & X_{t-2}(u_2) & \dots & X_{t-p}(u_2) \\ \vdots & \vdots & \ddots & \vdots \\ X_{t-1}(u_n) & X_{t-2}(u_n) & \dots & X_{t-p}(u_n) \end{pmatrix} \begin{pmatrix} \phi_{t,1} \\ \phi_{t,2} \\ \vdots \\ \phi_{t,p} \end{pmatrix} + \begin{pmatrix} \epsilon_t(u_1) \\ \epsilon_t(u_2) \\ \vdots \\ \epsilon_t(u_n) \end{pmatrix}$$

Random walk for the TVAR coefficients,

$$\Phi_t = \begin{pmatrix} \phi_{t,1} \\ \phi_{t,2} \\ \vdots \\ \phi_{t,p} \end{pmatrix} ; \Phi_t = \Phi_{t-1} + w_t$$

Gaussian Stochastic Process For the TVAR innovations

- Gaussian Random Field for ϵ_t:

$$\epsilon_t(\cdot) \sim \text{GP}(0, v_t c(\cdot, \cdot))$$

For finite observations specifically,

$$\begin{pmatrix} \epsilon_t(u_1) \\ \epsilon_t(u_2) \\ \vdots \\ \epsilon_t(u_n) \end{pmatrix} \sim \text{MVN}(0, V_t \times \Sigma(u_1, \ldots, u_n))$$

- Power Exponential Family of spatial correlation:

$$c(u, u^{'}) = \exp(-\beta \mid u - u^{'} \mid), \Sigma_{i,j} = c(u_i, u_j)$$

Discounting Variances

- Discount factor δ_1 for v_t to allow its stochastic changes,

$$v_t^{-1} \mid \mathrm{D}_{t-1} \sim \mathrm{G}(\delta_1 n_{t-1}/2, \delta_1 d_{t-1}/2)$$

choose $n_0 = 1, d_0 = \mathrm{var}(X)$.

- δ_2 for C_t,

$$w_t \sim \mathrm{MVN}(m_t, C_t)$$

$$C_t \mid \mathrm{D}_{t-1} = (1 - \delta_2) C_{t-1}/\delta_2$$

$$C_{t-1} = \mathrm{Cov}(\Phi_{t-1} \mid \mathrm{D}_{t-1})$$

choose $m_0 = 0, C_0 = 10 I_{p \times p}$

DLM Representation

$$(F, G, V, W)_t = (F_t, G_t, V_t, W_t)$$

$$F_t' = \begin{pmatrix} X_{t-1}(u_1) & X_{t-2}(u_1) & \dots & X_{t-p}(u_1) \\ X_{t-1}(u_2) & X_{t-2}(u_2) & \dots & X_{t-p}(u_2) \\ \vdots & \vdots & \ddots & \vdots \\ X_{t-1}(u_n) & X_{t-2}(u_n) & \dots & X_{t-p}(u_n) \end{pmatrix}$$

$$G_t = I_{p \times p}$$

$$V_t = v_t \Sigma(u_1, \dots, u_n)$$

Finally, v_t and W_t are sequentially specified.

Backward Sampling

Gibbs Sampler

Interested in: $(\{v_1, \ldots v_T\}; \{\Phi_1, \ldots, \Phi_T\}; \{\beta\} \mid \mathrm{D}_T)$

- Sample $(\beta \mid D_T, v_{1:T}, \Phi_{1:T})$.
- Sample $(v_{1:T}, \Phi_{1:T} \mid D_T, \beta)$.

Sample $(v_{1:T} \mid D_T, \beta)$.

Sample $(\Phi_{1:T} \mid v_{1:T}, D_T, \beta)$.

- Sample $(V_t, t = 1, \ldots, T \mid D_T, \beta)$:

  (a.) Forward filtering with unknown variances.
  (b.) Sample $(V_T^{-1} \mid D_T, \beta) \sim \mathrm{G}(n_T/2, d_T/2)$.

  (c.) Recursively sample $v_t, t = T-1, \ldots, 1$ from,

$$v_t^{-1} = \delta_1 v_{t+1}^{-1} + \mathrm{G}\left((1-\delta_1)n_t/2, d_t/2\right)$$

- Sample $(\Phi_{1:T} \mid D_T, v_{1:T}, \beta)$:

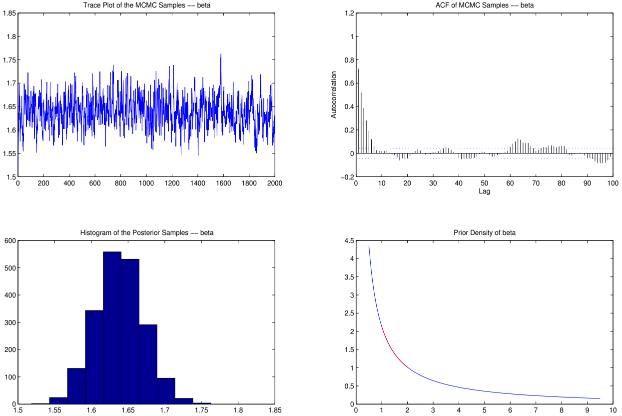
  (a.) Forward filtering again with known $v_{1:T}$.

  (b.) Sample $(\Phi_T \mid D_T, v_{1:T}) \sim \mathrm{MVN}(m_T, C_T)$.

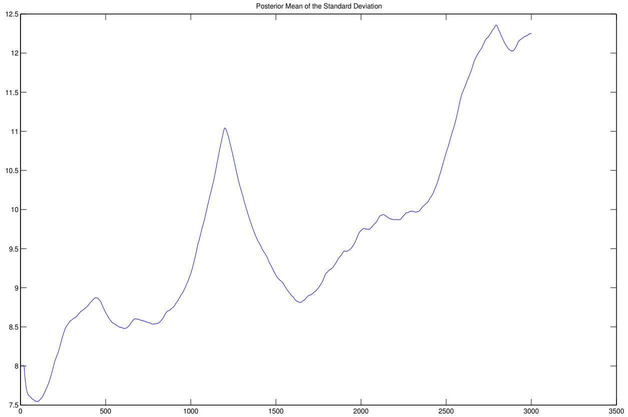
  (c.) Recursively sample $\Phi_t, t = T-1, \ldots, 1$ from,

$$(\Phi_t \mid D_T, \Phi_{t+1}, V_{1:T}) \sim \mathrm{MVN}\left((1-\delta_2)m_t + \delta_2\Phi_{t+1}, (1-\delta_2)C_t\right)$$

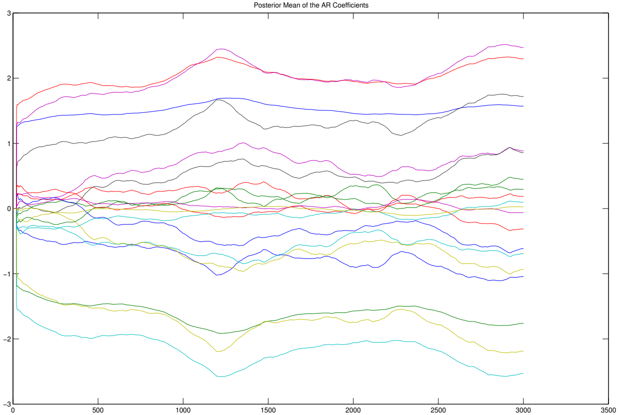
Posterior Distribution of β



Posterior Distribution of v_t



Posterior Distribution of ϕ



Spatial Interpolation —-
Predict Output of a computer model with new input

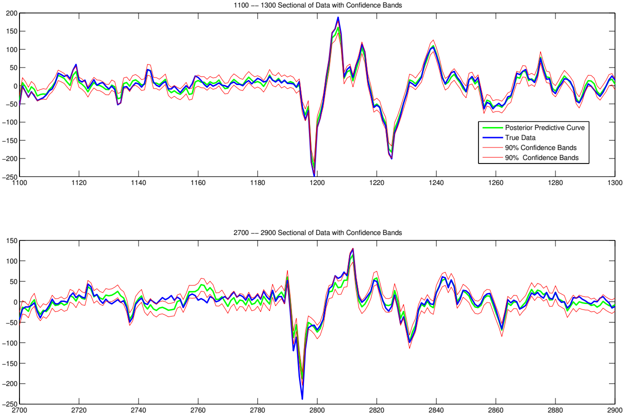
At new input u, we can predict (approximate) the computer model output its from the posterior draws.

$$\left(x_t(u) \mid x_{t-1:t-p}(u), \text{Data}, \Phi_{1:T} v_{1:T}, \beta\right) \sim \text{N}(\mu_t(u), \sigma_t^2(u))$$

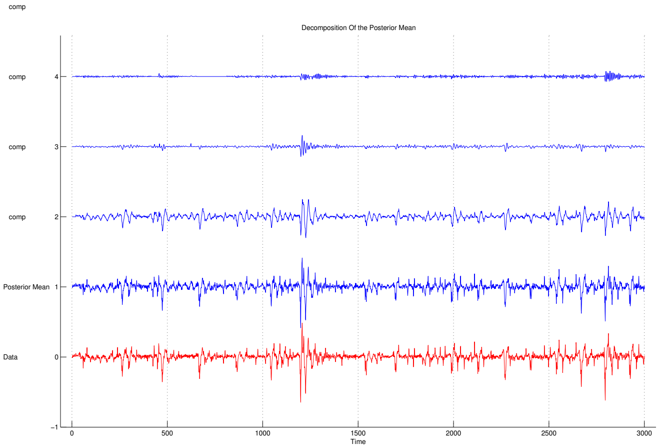
$$\mu_t(u) = \sum_j x_{t-j}(u)\phi_{t,j} + \rho^t(u, u_{1:n})\Sigma^{-1}(u_{1:n}, \beta) \begin{pmatrix} \epsilon_t(u_1) \\ \epsilon_t(u_2) \\ \vdots \\ \epsilon_t(u_n) \end{pmatrix}$$

$$\sigma_t^2(u) = V_t(1 - \rho^t(u, u_{1:n})\Sigma^{-1}(u_{1:n}, \beta)\rho(u, u_{1:n}))$$

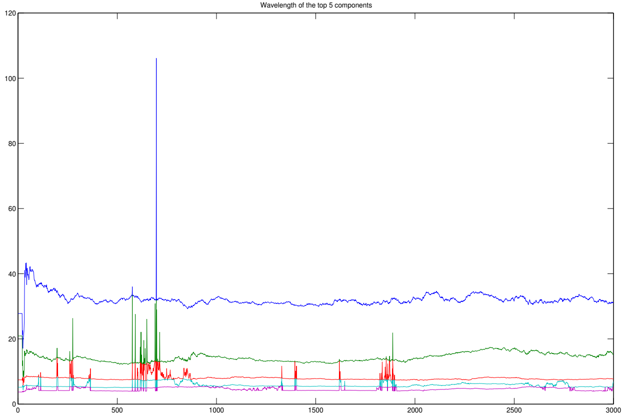
Predictive Curve with Posterior Quantiles



Decomposition – Posterior Mean



Wave Lengths – Posterior Mean



Moduli – Posterior Mean