Calibration of computational models of cerebral blood flows

Overview and goals

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Complex Computer Models, SAMSI, 2006-07

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Outline

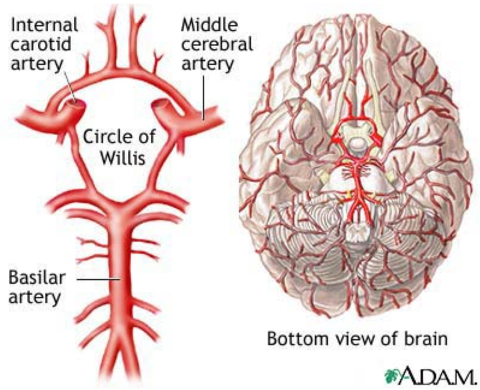
Who we are

- A. Cintron-Arias, Postdoc, SAMSI
- K. DeVault, Graduate Student, Math-NCSU
- P. Gremaud, Professor, Math-NCSU
- D. Maniyar, Graduate Student, Aston University
- M. Olufsen, Ass. Prof, Math-NCSU
- G. Vernieres, Postdoc, SAMSI
- D. Wilkinson, Professor, Newcastle University

Outline

1. The working group
2. Research goals
   - Background
   - Current model
   - Expected outcome
3. Methodology issues

Circle of Willis (CoW)



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Medical issues

- blood pressure change $\overset{\text{autoregulation}}{\Leftrightarrow}$ constant blood flow
- CoW structurally complete in only 50% of healthy brains
- autoregulation impairement $\Rightarrow$ stroke
- autoregulation impairement $\overset{?}{\Leftrightarrow}$ network topology

Modeling issues

- impossible to treat the whole vascular system at once $\Rightarrow$ how much of it to consider?
- how to link the computed part to the rest? boundary conditions
- how detailed should the model be? (3D throughout not doable)
- how to calibrate the model to the partial available data?

Outline

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Simplifying assumptions

- the blood density is constant,
- the blood flow is axisymmetric and no has no swirl,
- the vessels are tethered in their longitudinal direction,
- the equations are averaged on cross-sections.

Not so simplifying assumptions

- blood is considered as a non-Newtonian fluid,
- the effect of gravity is taken into account,
- a whole family of velocity profiles can be considered,
- viscoelastic reactions are included.

The equations

conservation of mass, momentum and viscoelastic model

$$(\partial_t + B\,\partial_x)\begin{bmatrix} A \\ Q \\ P \end{bmatrix} = \begin{bmatrix} 0 \\ -\frac{\gamma+2}{\mathcal{R}}\mu\frac{Q}{A} - \frac{e_x\cdot k}{\mathcal{F}}A \\ \frac{1}{\mathcal{W}}(1-P) + \frac{2}{\mathcal{W}\mathcal{M}^2}(1-A^{-1/2}) \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{\gamma+2}{\gamma+1}\left(\frac{Q}{A}\right)^2 & 2\frac{\gamma+2}{\gamma+1}\frac{Q}{A} & A \\ 0 & \frac{\tau_\epsilon}{\tau_\sigma}\frac{1}{\mathcal{M}^2}A^{-3/2} & 0 \end{bmatrix},$$

A, Q, P = surface area, flux, pressure; $\mathcal{F}, \mathcal{M}, \mathcal{R}, \mathcal{W}$ Froude, Mach, Reynolds, Weissenberg numbers

Mathematical structure

- 16 vessels
- 3 nonlinear balance laws per vessel
- vessels linked by boundary conditions (pressure continuity, conservation of flux)

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Expected outcome

“easy"	portable, fast, robust, easy to use (by MDs) computational tool for blood flow prediction in CoW
hard	reliable

Questions

Coupled multidomain models

- what can/should be tuned and what should not?
  - boundary conditions (for instance resistance model $P = QR$)
  - speed profile parameter γ ($\gamma = 2$: Poiseuille)
  - other things?
- how to include variations in one individual? group of individuals?