Quantifying Elephant Social Structure

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Elephant Social Structure

- Only females and juveniles form families. Adult males just run around looking to mate.
- The oldest female tends to be the leader since she's the largest and wisest.
- Elephants within a family are often related.

Scientific Questions

- What factors influence the development and evolution of social structure in elephants?
- What role does kinship play in the social structure?
- How does the social structure change in the Wet Season vs. the Dry Season?

Data Collection

- Biologists in Kenya ride into the National Park looking for herds of elephants.
- The adult female elephants are identified and recorded.
- The biologists either stay to observe the family or move on to find another herd.

Binomial Data Example

The total number of times data were recorded:

$$\text{Dry Season:} \quad N_{\text{Dry}} = 418$$
$$\text{Wet Season:} \quad N_{\text{Wet}} = 219$$

If Amy, Ang, and Aud are observed together, and the others are missing, then:

$$y_{\text{AmyAng}} = y_{\text{AmyAud}} = y_{\text{AngAud}} = 1$$
$$n_{\text{AmyAng}} = n_{\text{AmyAud}} = n_{\text{AngAud}} = 1$$

While for one missing elephant:

$$y_{\text{AmyAga}} = 0$$
$$n_{\text{AmyAga}} = 1$$

Whereas for two missing elephants:

$$y_{\text{AliAga}} = 0$$
$$n_{\text{AliAga}} = 0$$

Binomial Data, Cont.

In this example of five elephants Amy, Angelina, Audrey, Alison, and Agatha at time $= t$, the $\mathbf{y}$ matrix of successful observations would be:

$\mathbf{y_t} =$

	Amy	Ang	Aud	Ali	Aga
Amy	$\ddots$	1	1	0	0
Ang	1	$\ddots$	1	0	0
Aud	1	1	$\ddots$	0	0
Ali	0	0	0	$\ddots$	**0**
Aga	0	0	0	**0**	$\ddots$

The $\mathbf{n_t}$ matrix of potential observations =

	Amy	Ang	Aud	Ali	Aga
Amy	$\ddots$	1	1	1	1
Ang	1	$\ddots$	1	1	1
Aud	1	1	$\ddots$	1	1
Ali	1	1	1	$\ddots$	**0**
Aga	1	1	1	**0**	$\ddots$

The Model

- Data are binomial observations on pairs of elephants
  - $y_{ij} \sim \text{Bin}(n_{ij}, p_{ij})$
  - y_{ij} is the number of times elephants i and j observed together.
  - n_{ij} is the number of times either i or j observed.

- Use a Generalized Linear Model $\Longrightarrow$ Logistic regression
  - $E(y_{ij}|\,\theta_{\mathbf{ij}}) = g(\theta_{\mathbf{ij}})$.
  - g is the inverse logit link function.
  - The probability of elephants i and j being together is:

$$p_{ij} = \frac{\exp\theta_{\mathbf{ij}}}{1+\exp\theta_{\mathbf{ij}}}.$$

- θ_{ij} ***is the linear predictor.***

Linear Predictor θ_{ij}

How often are two elephants together?

- Common intercept β_0, a baseline probability.

- Intrinsic sociability a_i random effect.
  - Sociable elephants will more often be observed in large groups.

- Kinship relatedness $\beta_k \, k_{ij}$.

  1- DNA relatedness measure k_{1ij}: how closely elephants i and j are related

  2- Mother/Daughter pair indicator k_{2ij}

  3- Sisters pair indicator k_{3ij}

- Normal error γ_{ij} (unexplained error or white noise).

- **Pairwise effect** $z_i' z_j$ between elephants i and j.

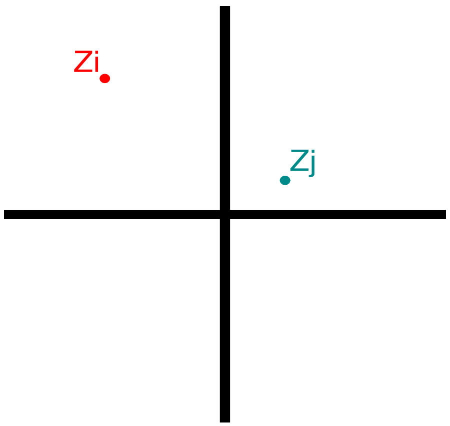
$$\theta_{\mathbf{ij}} = \beta_{\mathbf{0}} + \mathbf{a_i} + \mathbf{a_j} + \beta_{\mathbf{k}} \, \mathbf{k_{ij}} + \gamma_{\mathbf{ij}} + \mathbf{z_i' z_j}$$

Pairwise Effects

$z_i' z_j$ is the inner product of the positions of elephants i and j in latent (unobserved) Social Space.

- I choose the dimension of social space $k = 2$.

Elephants i and j have positions $\mathbf{z}_i$ and $\mathbf{z}_j$ in 2D social space.



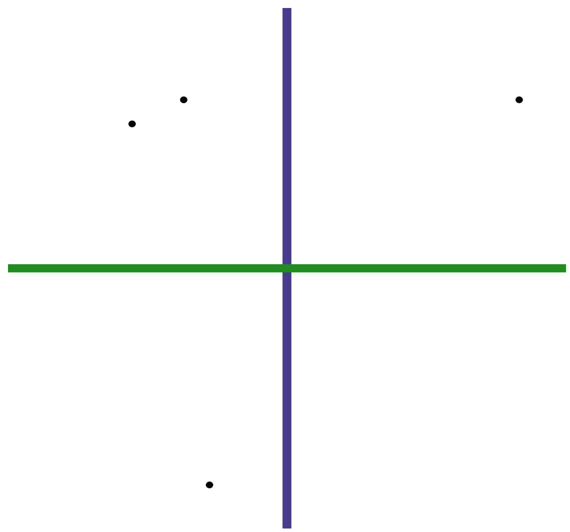
$$\mathbf{z}_i \sim N(\mathbf{0}, \sigma_z^2\,\mathrm{I})$$
$$\mathbf{z}_j \sim N(\mathbf{0}, \sigma_z^2\,\mathrm{I})$$

- If $\mathbf{z_i' z_j} = \mathbf{0}$ then elephants i and j interact as often as their sociabilities a_i, a_j and their kinships k_{ij} would predict.
- If $\mathbf{z_i' z_j} > \mathbf{0}$ then i and j like each other and are observed together more often than the model would otherwise predict.
- If $\mathbf{z_i' z_j} < \mathbf{0}$ then i and j dislike each other.

Social Space Example

2 "most important" directions of compatibility

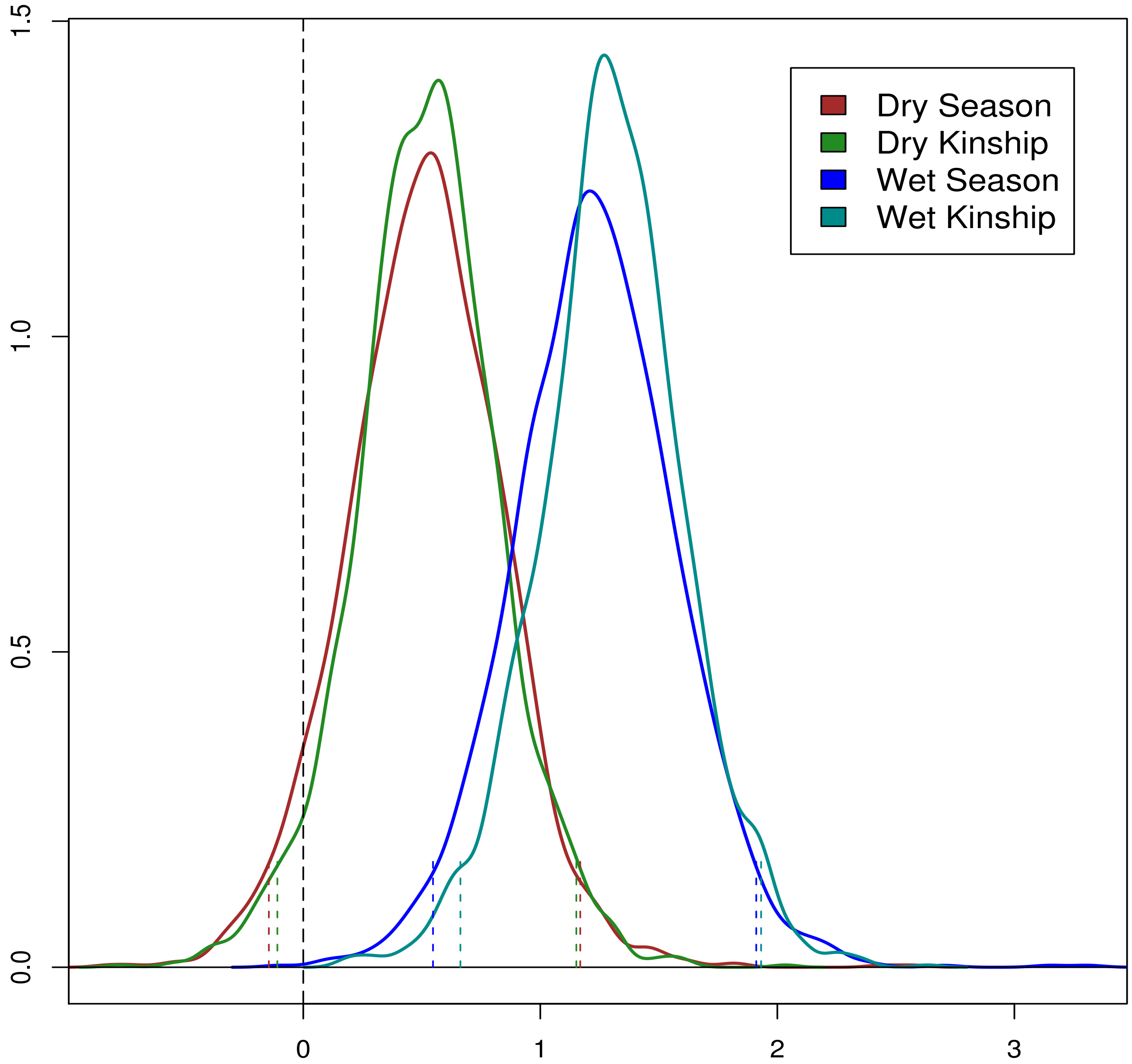
- Attitude towards money: Frugal ⟵ – – – – – – – – ⟶ Extravagant
- Sleep schedule: Early bird ⟵ – – – – – – – ⟶ Night owl
- Combine both to create a 2D Social Space



- The inner product of two vectors is the similarity of their directions, scaled by their lengths.

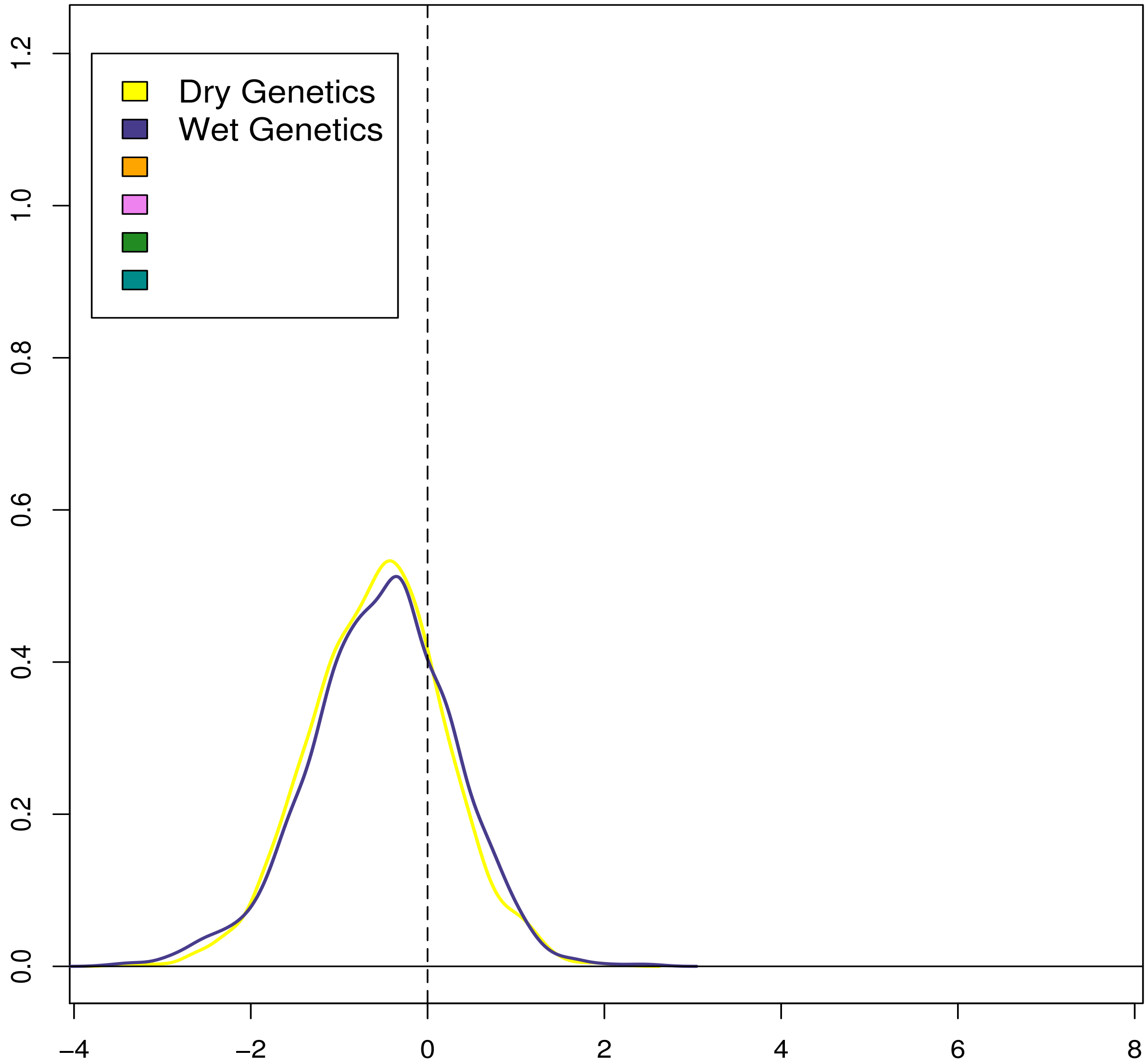
Elephant Family Results

Posterior Intercepts β_0



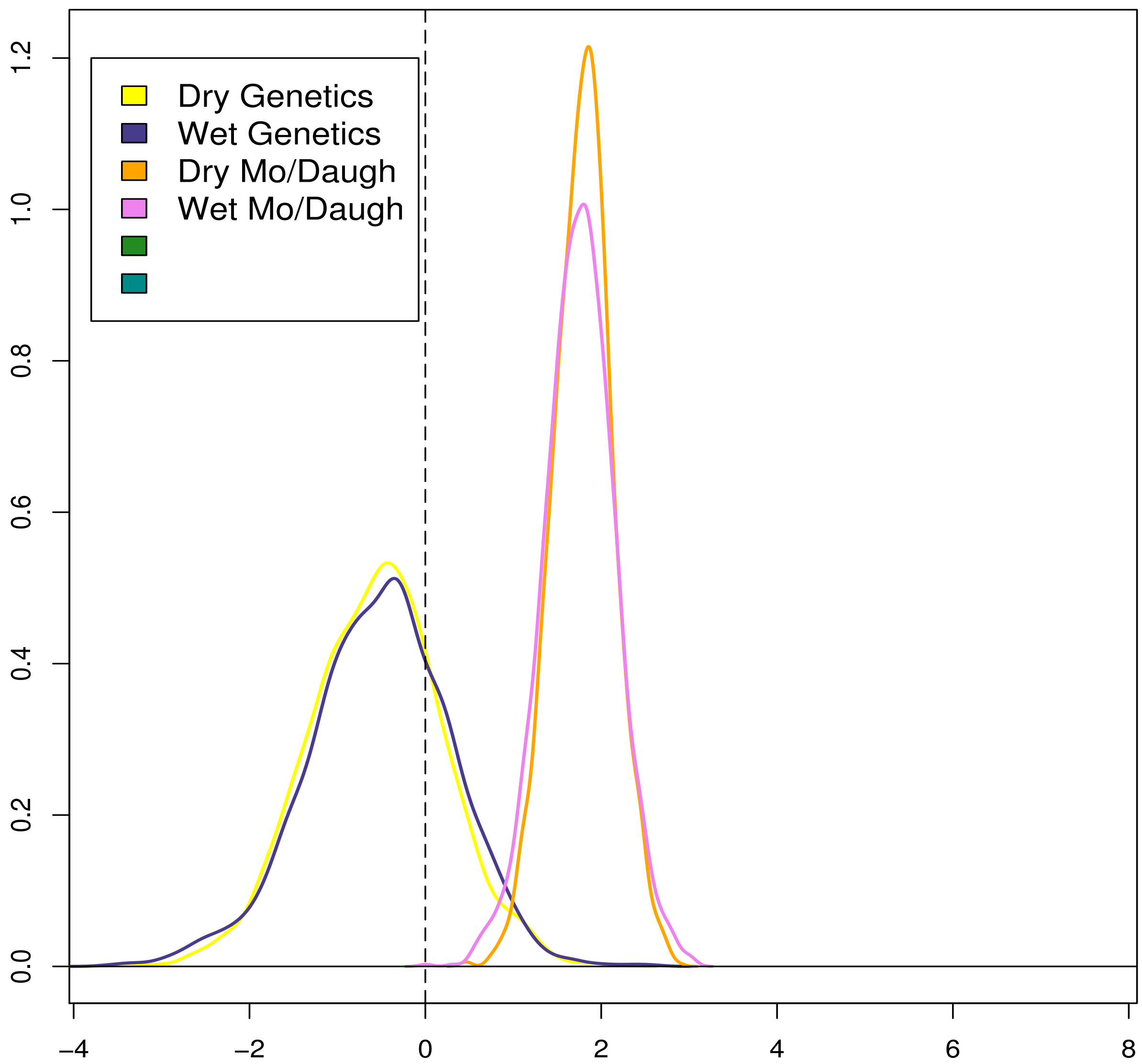
$$\theta_{ij} = \beta_0 + a_i + a_j + \beta_k k_{ij} + \gamma_{ij} + z_i' z_j$$

Posteriors for Kinship Coefficients $\beta_{\mathbf{k}}$



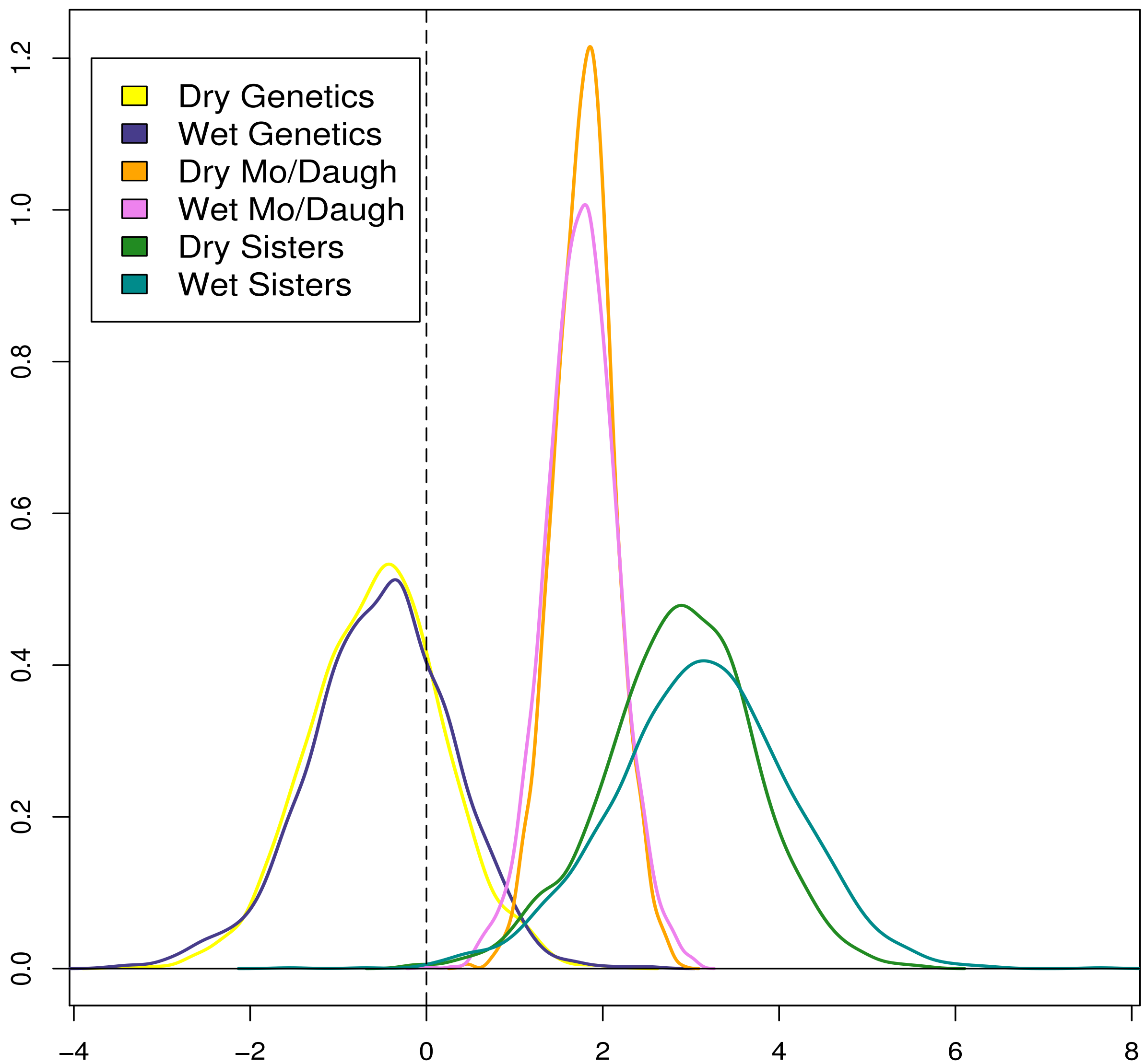
$$\theta_{ij} = \beta_0 + \mathbf{a_i} + \mathbf{a_j} + \beta_{\mathbf{k1}}\,\mathbf{k_{1ij}} + \beta_{\mathbf{k2}}\,\mathbf{k_{2ij}} + \beta_{\mathbf{k3}}\,\mathbf{k_{3ij}} + \gamma_{\mathbf{ij}} + \mathbf{z_i'z_j}$$

Posteriors for Kinship Coefficients β_k



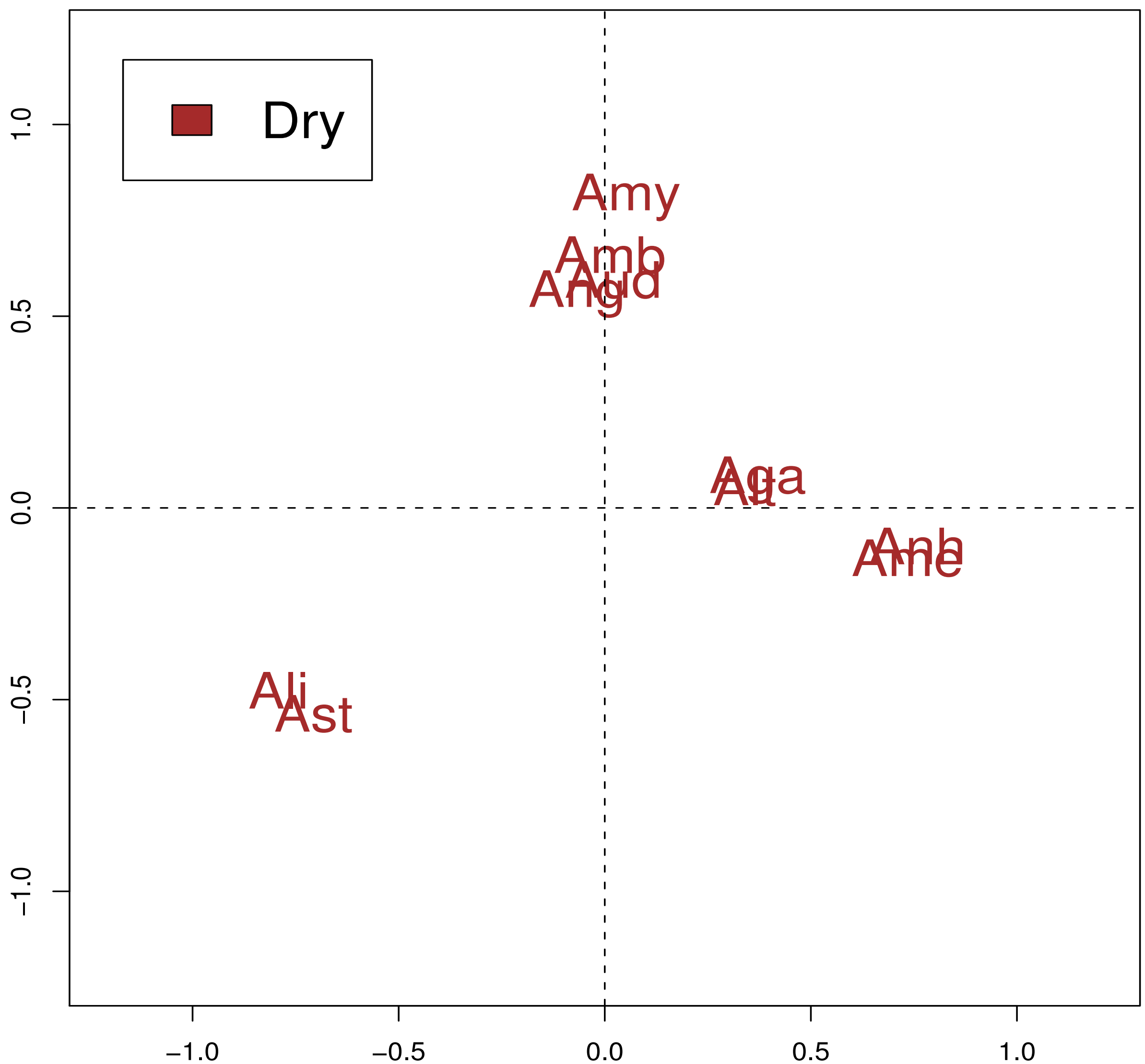
$$\theta_{ij} = \beta_0 + a_i + a_j + \beta_{k1}\, k_{1ij} + \beta_{k2}\, k_{2ij} + \beta_{k3}\, k_{3ij} + \gamma_{ij} + z_i' z_j$$

Posteriors for Kinship Coefficients β_k



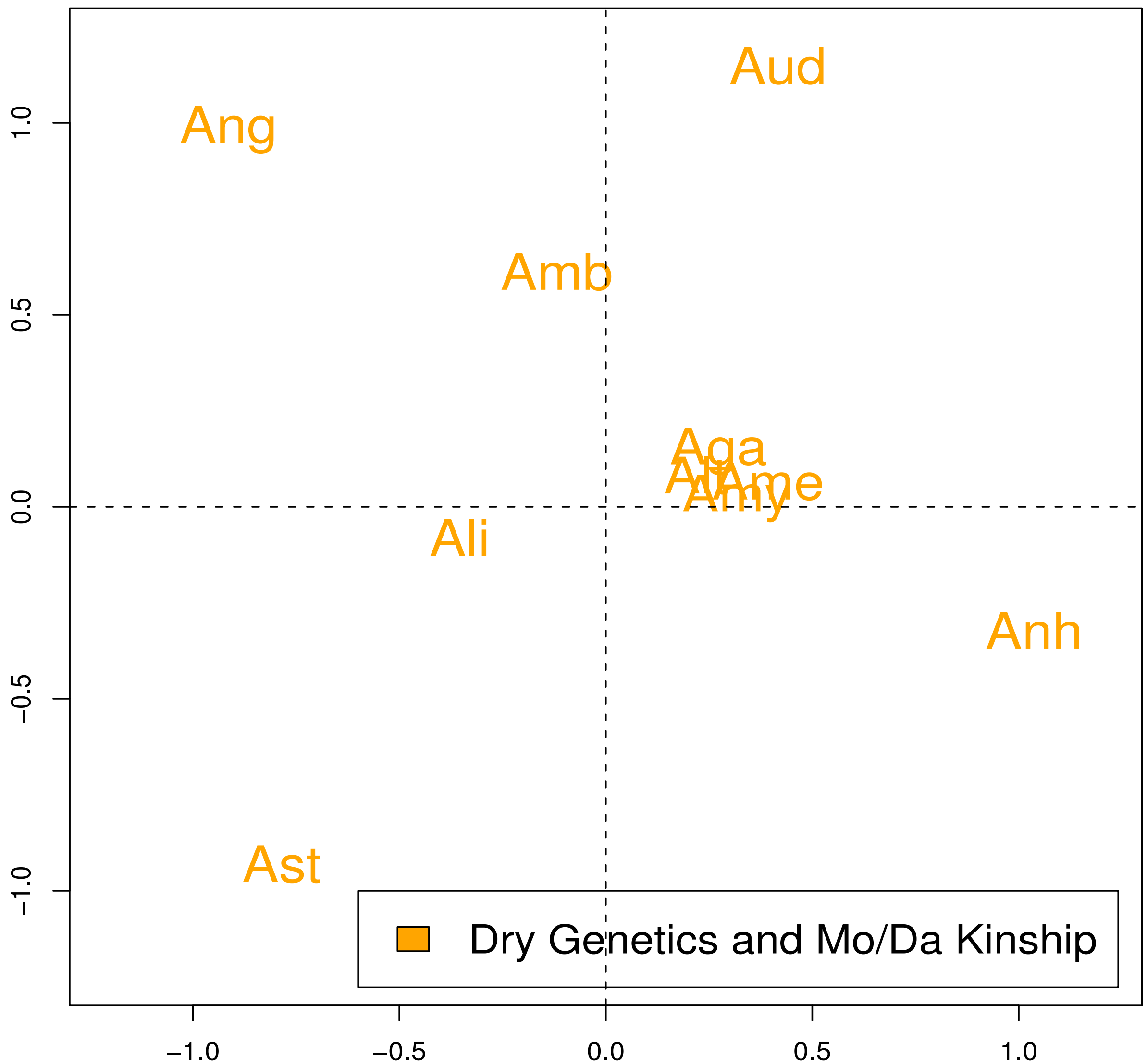
$$\theta_{ij} = \beta_0 + a_i + a_j + \beta_{k1}\, k_{1ij} + \beta_{k2}\, k_{2ij} + \beta_{k3}\, k_{3ij} + \gamma_{ij} + z_i' z_j$$

Dry Season Social Space $\bar{\mathbf{z}}_\mathbf{i}$ Posteriors



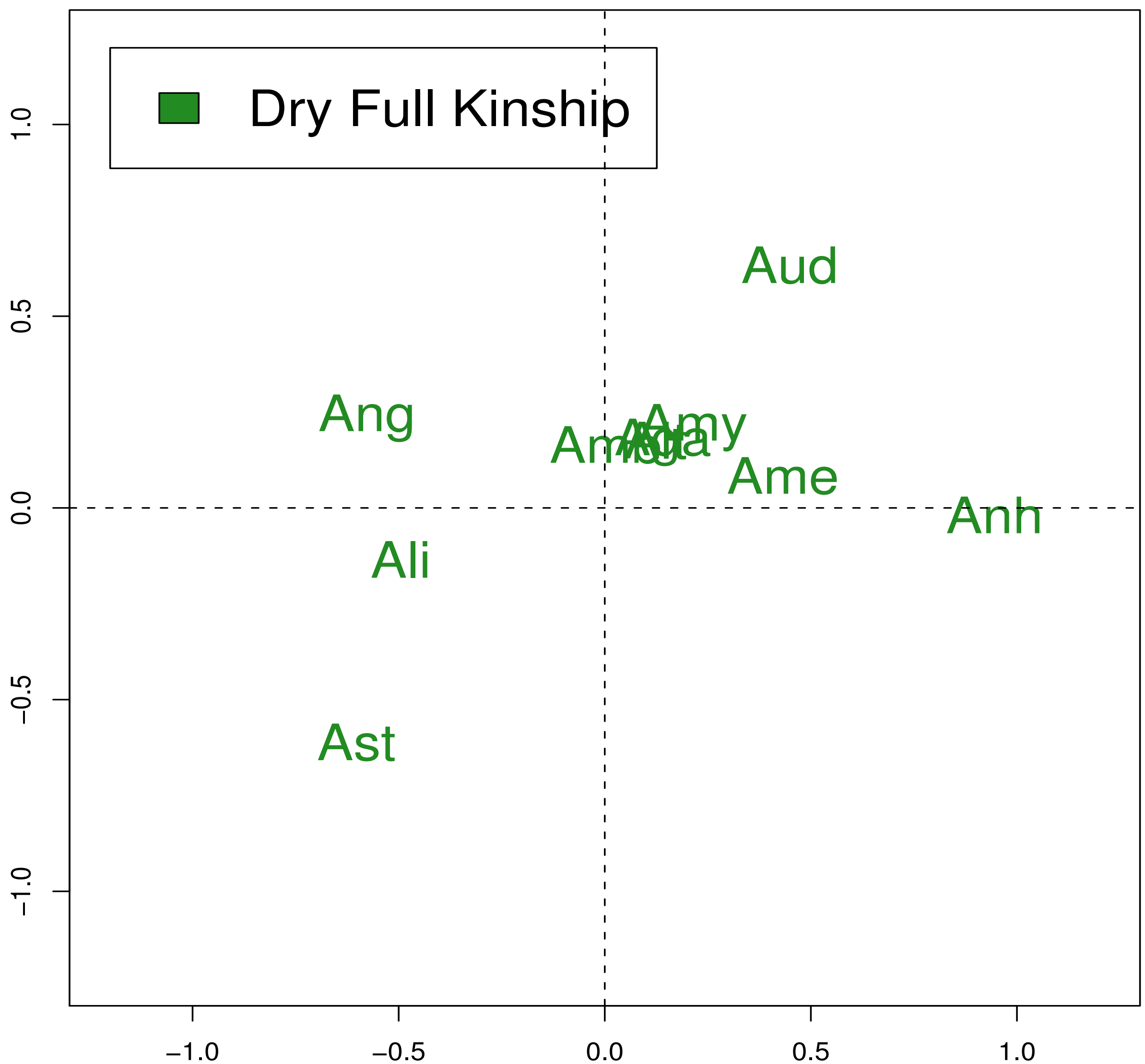
$$\theta_{\mathbf{ij}} = \beta_\mathbf{0} + \mathbf{a_i} + \mathbf{a_j} + \gamma_{\mathbf{ij}} + \mathbf{z_i'z_j}$$

Dry Gen, Mother/Daughter Social Space $\bar{\mathbf{z}}_\mathbf{i}$ Posteriors



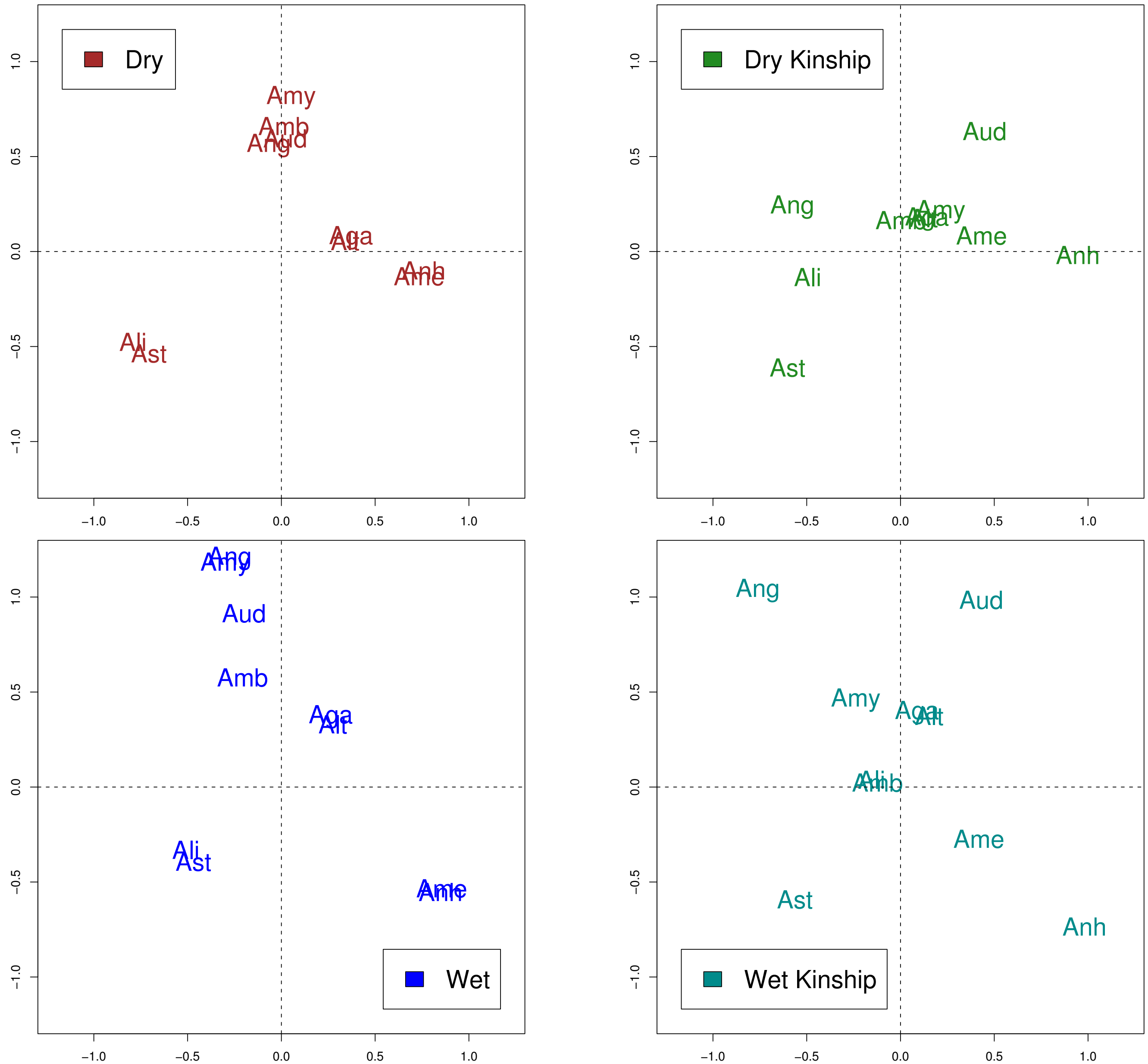
$$\theta_{ij} = \beta_0 + a_i + a_j + \beta_{k1}\, k_{1ij} + \beta_{k2}\, k_{2ij} + \gamma_{ij} + z_i' z_j$$

Dry Full Kinship Social Space $\bar{\mathbf{z}}_i$ Posteriors



$$\theta_{ij} = \beta_0 + \mathbf{a}_i + \mathbf{a}_j + \beta_{k1}\,\mathbf{k}_{1ij} + \beta_{k2}\,\mathbf{k}_{2ij} + \beta_{k3}\,\mathbf{k}_{3ij} + \gamma_{ij} + \mathbf{z}_i'\mathbf{z}_j$$

Social Space Posterior Means $\bar{z}_i$



Conclusions

- The intercept β_0 is greater in the Wet seasons than in the Dry seasons, indicating that the elephants are more gregarious during the Wet season.
- The role of kinships β_k is similar in the Wet and Dry season.
- Mother/Daughter relationships are significant.
- Sister relationships are also very important.
- After accounting for Mother/Daughter and Sister relationships, the DNA relatedness between elephants doesn't really matter.

Amy, Matriarch of Family AA