Friendships, IQ, Sex, and Social Status: Comparing Equation-based p-models with Intelligent Agent Models

Model 2 (Gender is added)

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1 Models and Rules

The basic p-friendship model:

$$\log(\frac{p_{ij}}{1-p_{ij}}) = \beta_0 + \beta_s\, s_i + \beta_r\, r_j + \delta\, X_{ij} - |z_i - z_j|$$

⋆ p-Model 2:

$$logit(p_{ij}) = \beta_0 + \delta_1 \left(\mathbf{I}(Sex_i = Sex_j) - \overline{\mathbf{I}(Sex_i = Sex_j)}\right) - |z_i - z_j|$$

- $logit(p_{ij})$ is the *degree* of friendship between agents i and j. p_{ii} is undefined.
- β_0 is the baseline degree of friendship between any two agents.
- $\mathbf{I}(Sex_i = Sex_j)$ is an indicator whether agents i and j are of the same **Sex**. This dyadic covariate is centered about its mean to retain the interpretation of the baseline degree of friendship β_0.
- z_i is the position of agent i in two-dimensional Social Space. $|z_i - z_j|$ is the distance between agents i and j.

⋆ Rules for Agent Model 2:

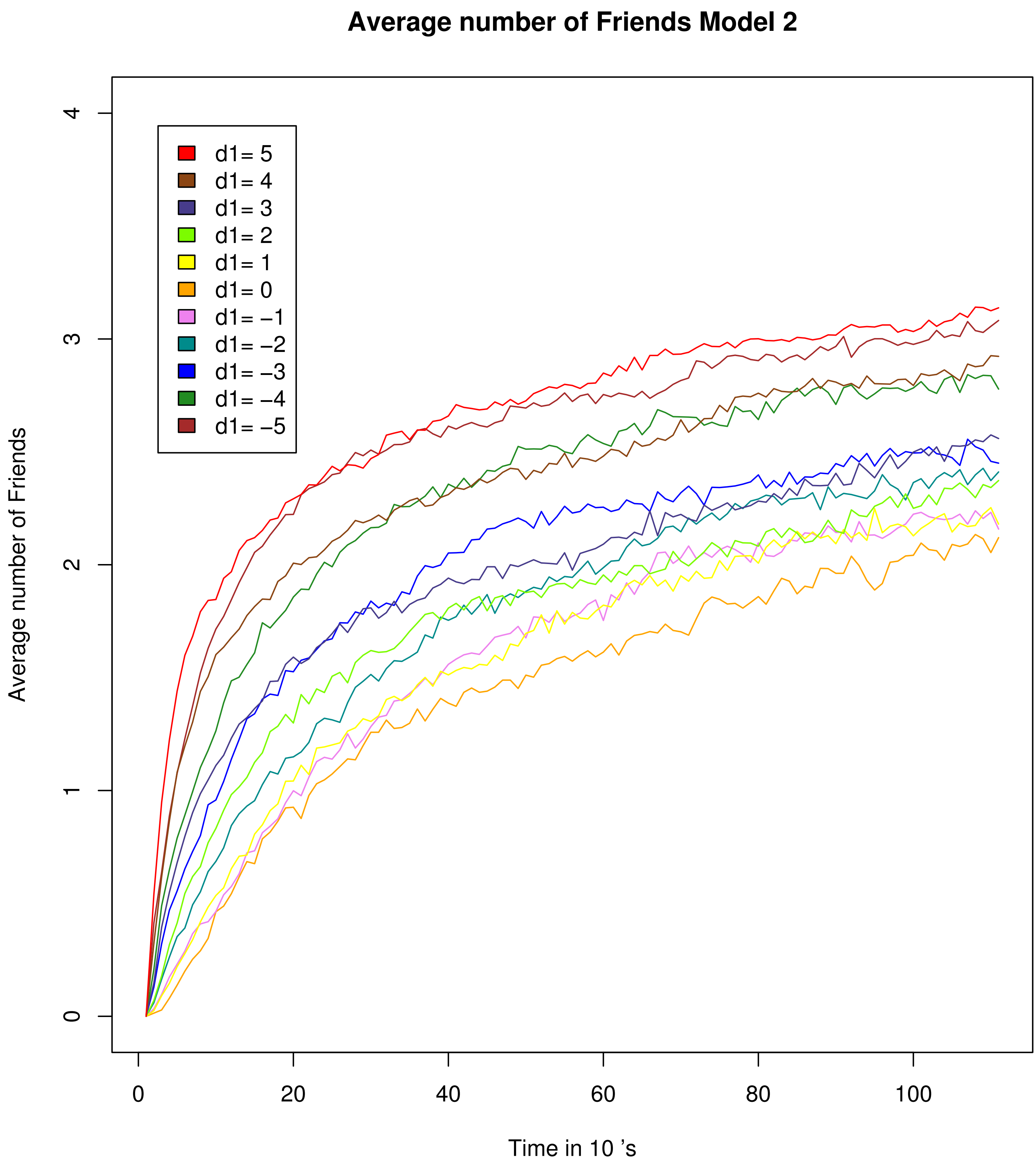
- Rule 0. Agents start randomly at time=1 on grid in 2-dimensional Social Space.
- Rule 1. Agents are split evenly between the genders. Charisma (sociality or extravertedness) remains absent.
- Rule 2. At every time step each agent i proffers a friendship to all agents $j \neq i$, and these proffers are accepted with probability p_{ij}.
- Rule 3. After new friendships are created, agents move a "**move.fraction**" towards the average of their friends' locations in Social Space. The size of Social Space is $n \times n$, (20×20).

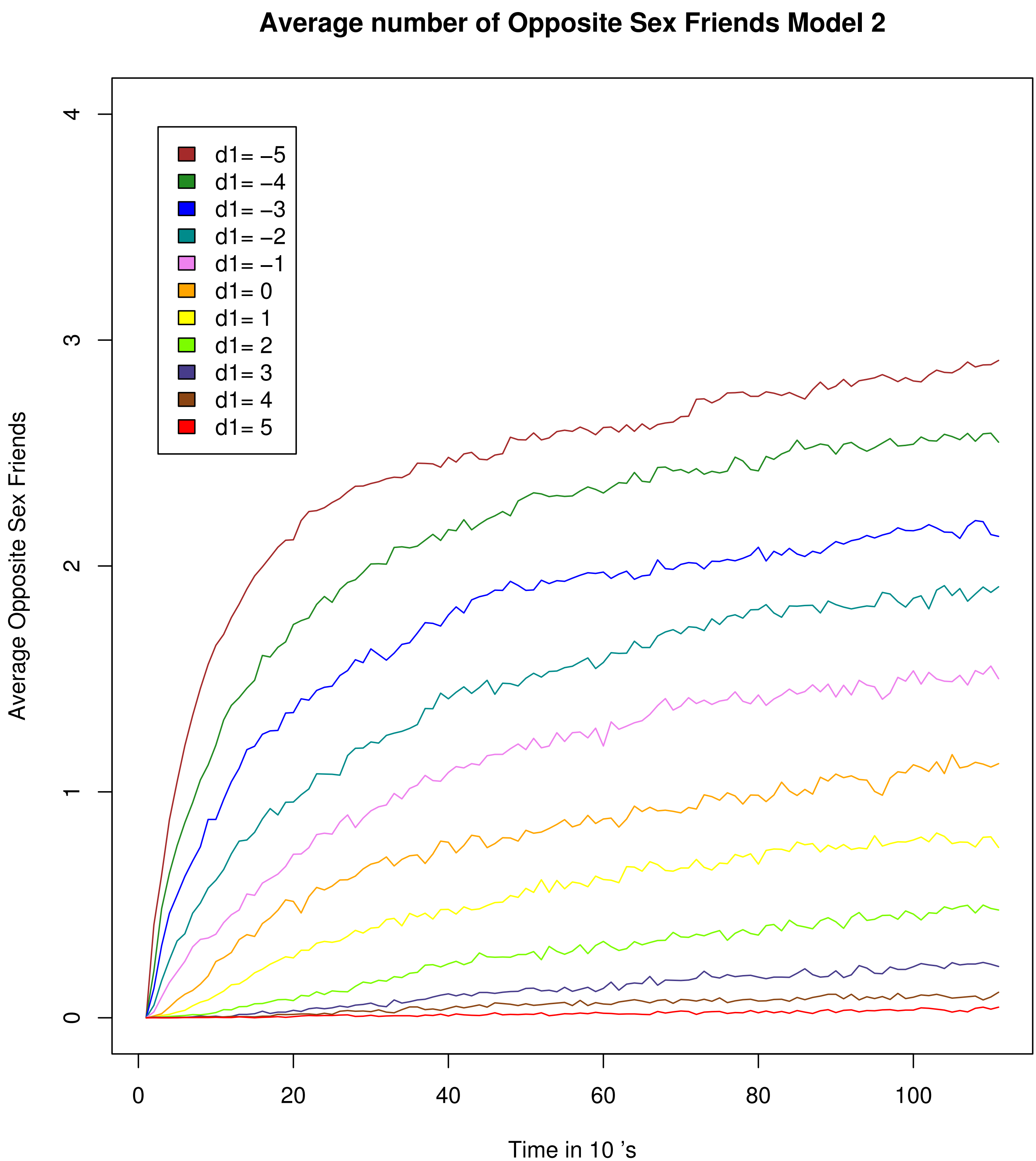
Implications of Rules for Model 2

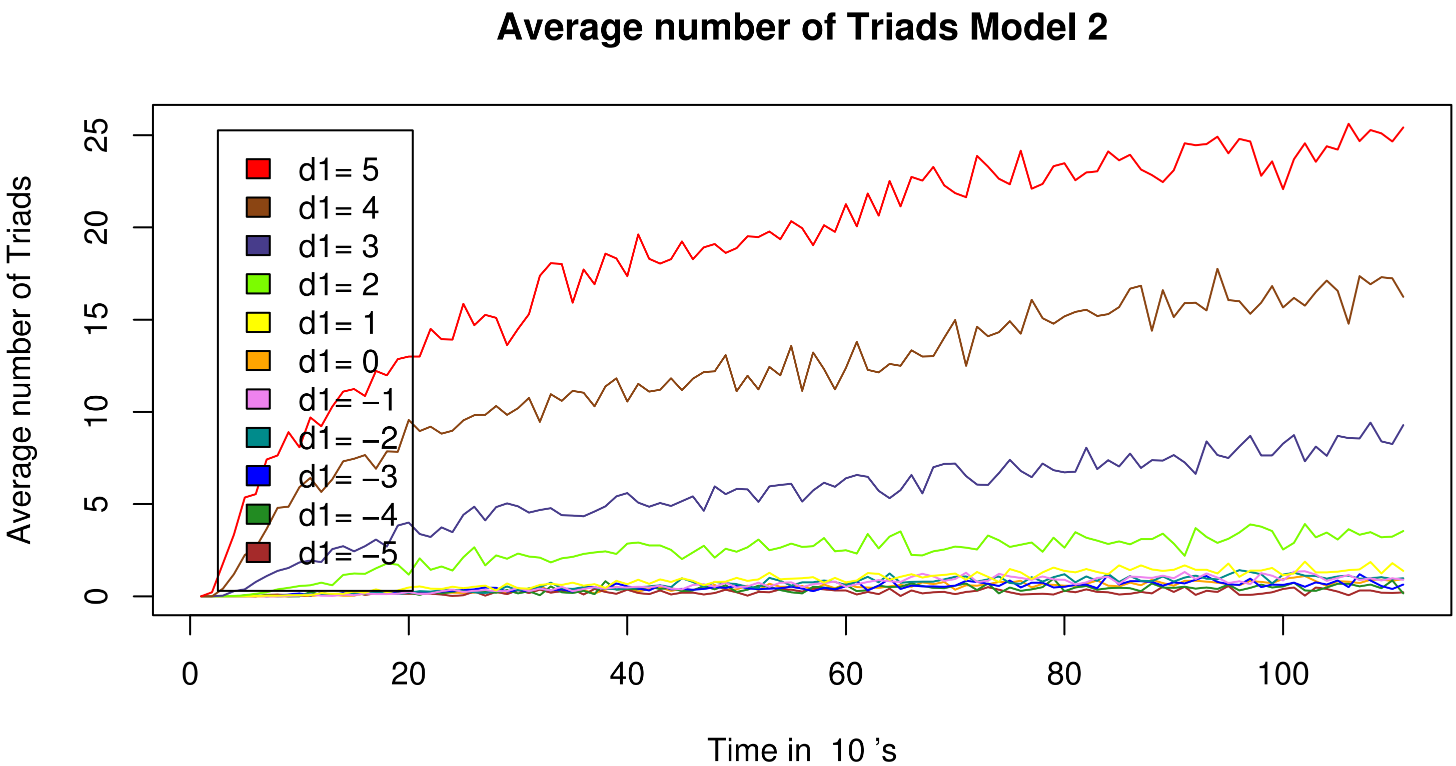
- β_0, **move.fraction**, δ_1, the Male/Female ratio, and the size of Social Space are potential parameters in Model 2. What is the appropriate size of Social Space? (in relation to the other parameters)
- Agents have a higher chance of becoming friends based on their Sex. The value of δ_1 determines if an agent is more or less likely to become friends with a member of the opposite sex.
- History, or memory, is absent from Models 2 and 1. What happens at time$= t$ only depends on times$< t$ through the movement in Social Space.
- Four summary statistics are: 1) Average number of Friends; 2) Number of completed Triads; 3) Number of very close Clusters; 4) Average number of Opposite Sex friends.

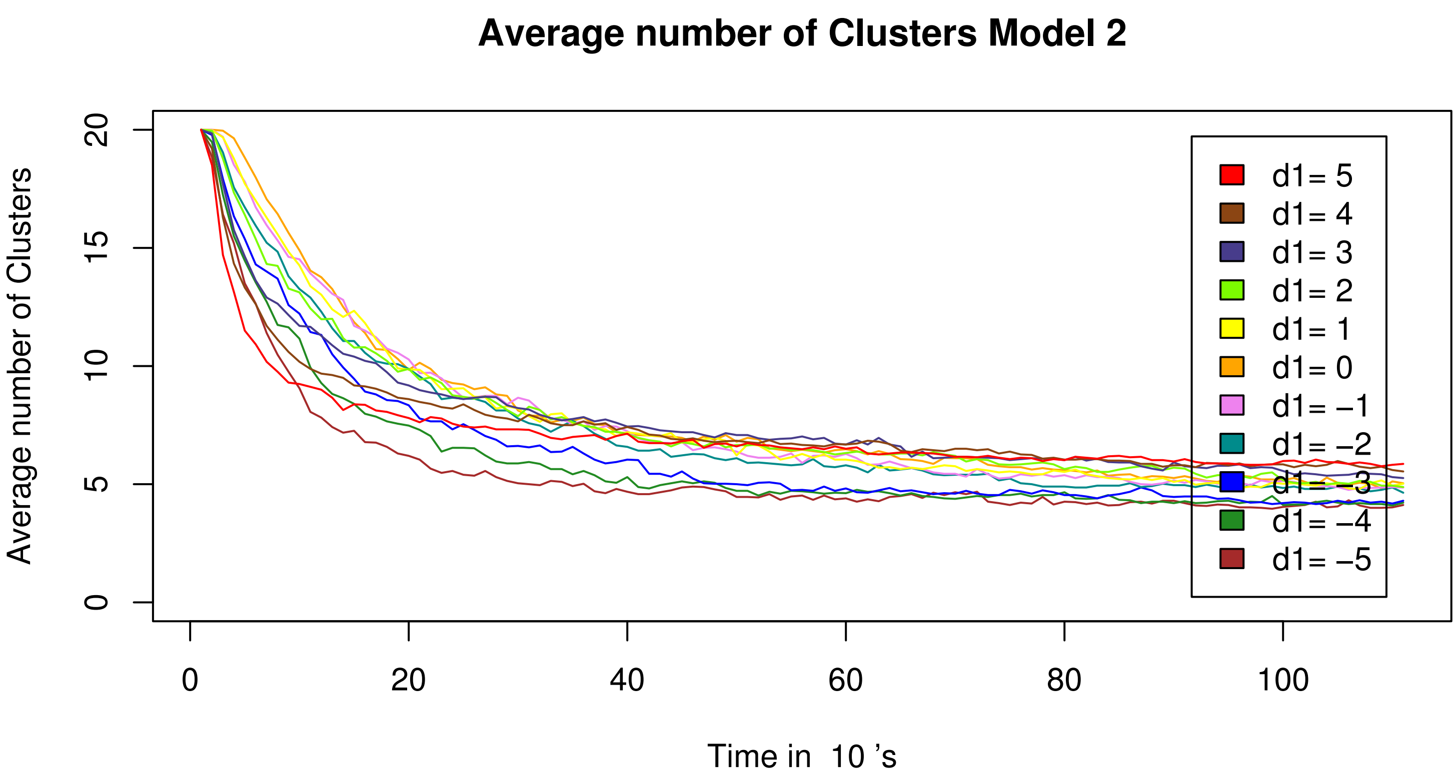
2 Results

For eleven values of the sensitivity to **Sex** parameter δ_1, I ran the intelligent agent model 50 times. I kept the summary statistics for every tenth time step then averaged them. These summary statistics are shown below.



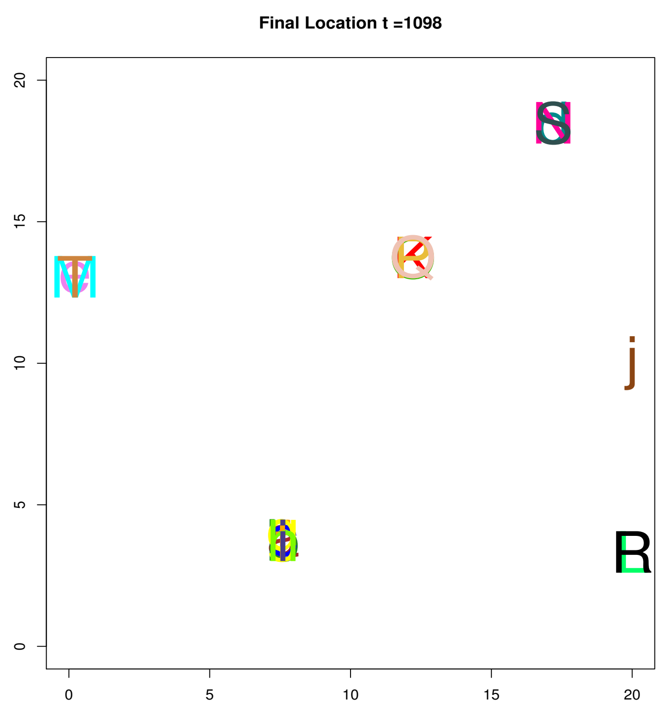

Average number of Opposite Sex Friends Model 2
Average Opposite Sex Friends
Time in 10 's
0
1
2
3
4
0
20
40
60
80
100
d1= −5
d1= −4
d1= −3
d1= −2
d1= −1
d1= 0
d1= 1
d1= 2
d1= 3
d1= 4
d1= 5


Average number of Triads Model 2
Average number of Triads
Time in 10 's
d1= 5
d1= 4
d1= 3
d1= 2
d1= 1
d1= 0
d1= −1
d1= −2
d1= −3
d1= −4
d1= −5
0
5
10
15
20
25
0
20
40
60
80
100


Average number of Clusters Model 2
Average number of Clusters
Time in 10 's
d1= 5
d1= 4
d1= 3
d1= 2
d1= 1
d1= 0
d1= −1
d1= −2
d1= −3
d1= −4
d1= −5
0
5
10
15
20
0
20
40
60
80
100

3 Results, Implications, Observations, and Notes for Model 2

- As long as **move.fraction** > 0, all the agents will eventually move together to form one perfect cluster. But it could be practially impossible based on β_0, δ_1, and the size of Social Space.
- Often one agent (starting in a corner of Social Space) will fail to make friends early and become isolated (nearly forever).
- Even when δ_1 was large, clusters of opposite sexes emerged in Social Space.
- Students in the same location in Social Space are not necessarily friends.
- In general, the larger δ_1 in absolute value, the higher the average number of friends.
- Males and Females lie on different "planes" in Social Space. $\delta_1 = 1$ implies that there is an extra unit of "distance" between Males and Females. What is the difference between the dyadic covariates and just a modification to Social Space? Answer: dyadic covariates are observed, whereas Social Space is not. Agents with lots of charisma could "warp" Social Space to make everyone seem closer. Could Social Space be made to be a relative construct, relative to i and j's **Sex**, charisma, and the baseline environment in which they live?
- An example of Social Space after 1098 iterations $\delta_1 = 2$ (preference for same sex):



Six clusters of students emerge: A big cluster of 7 males, a cluster of 4 females, a smaller cluster of 2 females, two mixed clusters of 1 male and 2 females, and one lone male who never made any friends.

- Why do we use Euclidean distance? Should there be a coefficient on the $|z_i - z_j|$ term?
- How does the ratio of males to females affect the dynamics?
- Adding sex into the equation drastically changes the dynamic behavior of the system. No longer do we observe one perfect cluster of agents at the end of the run. "Static" sub-clusters in Social Space form.

4 Notes on Future Work

- It should be easier for students to make friends at the beginning of the simulation, and harder as time goes by. This could be an another dynamic addition to the p-model. The β_0 baseline rate could decrease with time.
- Draw a graphs based on the Friendship matrix. Compare to Social Space.
- Define and identify "equilibrium states"?
- Make animations of dynamic Social Space.

5 Model 2 Longrun addition

⋆ p-Model 2:

$$logit(p_{ij}) = \beta_0 + \delta_1 \left(\mathbf{I}(Sex_i = Sex_j) - \overline{\mathbf{I}(Sex_i = Sex_j)} \right) - |z_i - z_j|$$

- I ran **Model 2** for 50 long runs of 10998 steps. Note the roughly stable number of clusters for each of the δ_1 (preference for same sex friends) values.