Agent-Based Methods for Dynamic Social Networks

Eric Vance

Institute of Statistics & Decision Sciences

Duke University

STA 395 Talk

October 24, 2005

Outline

- Introduction
- Social Network Models
- Agent-based Models
- Our Approach
- Results
- Statistical Challenges

Scientific Interest in Dynamic Social Networks

How do networks change over time?

How do we identify patterns?

How do we make predictions?

Examples:

- Babboon Fissions
- Antarctic Research Stations
- Terrorist Networks
- Elephants

Scientific Questions–Babboons

- Once in a lifetime (≈10 years) babboons will permanently fission from one group into two.
- Before the final fission babboons repeatedly split into “test” groups. After a few hours or days apart they come back together. Some time later they (temporarily) split again.
- What does this changing social structure tell us about how babboons make decisions?

Scientific Questions–Antarctic Researchers

- How could one build cohesiveness in an Antarctic research station?
- What early characteristics of the group will lead to "perfect" clustering, cliques, loners? Can these aspects of the social structure be predicted or changed by studying its dynamics?
- What is the effect of having a few charismatic personalities, or someone everybody hates?

Scientific Questions–Terrorists

- How will the structure of Al Qaeda look in the future?
- What could dynamic data tell us about characteristics of the network?
- How would the elimination of key individuals in the network affect the evolution of the network?

Scientific Questions–Elephants

- How do changing resources affect elephant social structure (Wet Season v. Dry Season)?
- Do elephants change their preferences over time? How might this affect the social structure?
- What effect might an elephant's death have on the group structure?

Models of Social Networks

Social network analysis models relationships between actors.

- The attributes of individual actors are not as important as the attributes of their relationships, or ties, with other actors.
  - Presence/absence of a friendship (0 or 1)
  - Money exchanged between actors ($0.55)
  - a Win, Loss, or Draw (1, -1, 0) in a competition.

- A social network can be represented as a graph where the nodes are the actors and the edges represent the ties between the actors.

- A social network can also be represented by its matrix of ties:

	Amy	Ang	Ali	Ast
Amy	0	1	0	0
Ang	1	0	0	0
Ali	0	0	0	1
Ast	0	0	1	0

Social Network Models

- Holland and Leinhardt (1981) p_1 model
  - Assumes independence of dyads
  - Models the ties as functions of individual relational attributes (expansiveness, attractiveness) as well as features of the graph (density, overall tendency toward reciprocity)
- Wasserman and Pattison (1986) p^* model
  - Models the probabilities of ties conditional on all other ties in the network.
  - Explanatory variables are the differences in the network statistics (...) if a tie x_{ij} were to change from 1 to 0.
- Hoff, Raftery, and Handcock (2002)

Hoff, Raftery, and Handcock Social Space Model

Similar to the model I use for elephants!

- Sender and receiver random effects, as well as positions in a latent social space, account for the dependence between dyads.

- $\text{logit}(p_{ij}) = \beta_0 + s_i + r_j + \beta_d X_{ij} - |z_i - z_j|$
  - Common intercept β_0, a baseline probability
  - Sender sociability or "expansiveness" s_i random effect
  - Receiver "attractiveness" r_j random effect
  - Vector of dyad-specific (observable) covariates X_{ij}

- The distance between i and j in "Social Space" affects the probability of a tie from $i \longrightarrow j$.
  – Actors close together in social space are more likely to form ties.

Agent-Based Models

- Impose a few simple rules on agents, then study the aggregate effects of the resulting interactions.
- Complex social phenomena can be generated by individual agents acting according to the simple rules.
- Sugarscape (Epstein and Axtell 1996) is a classic example.

Sugarscape Example

- Agents collect sugar according to "vision" rules and burn it at an individual rate called "metabolism".
- The Sugarscape has rules for where and how fast sugar regrows.
- Patterns of migration and skewed distributions of wealth emerge.

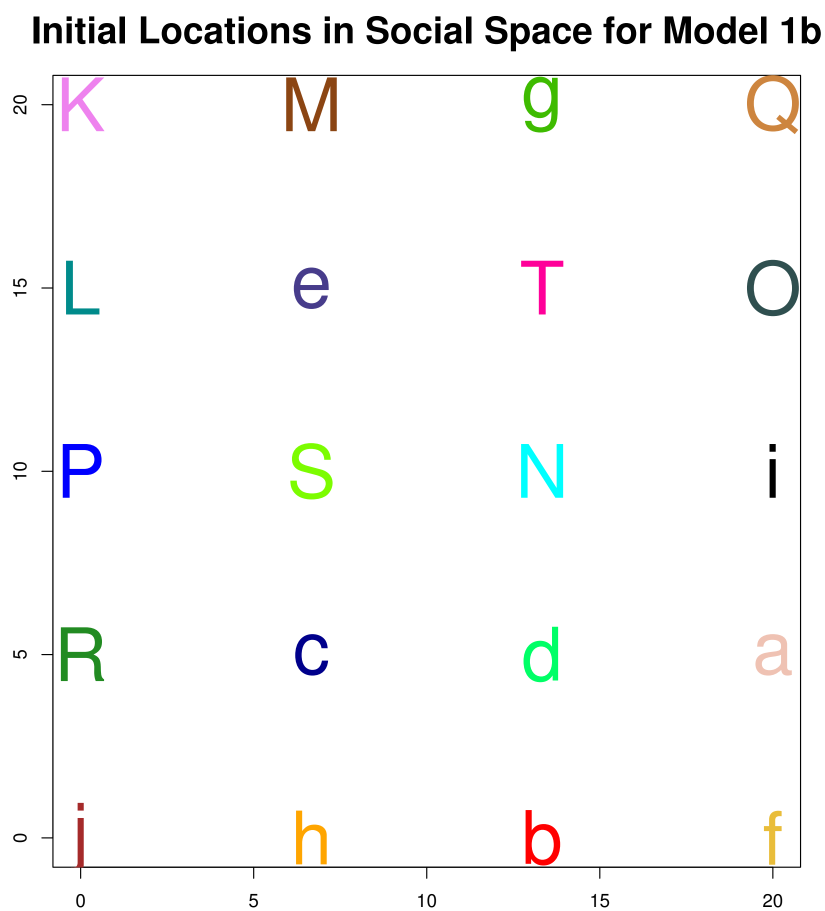
⋆ New rules produce new phenomena:

Rules	Implications
Vision, Metabolism, Regrowth rates	Migration, Wealth distribution
Sex, Reproduction	Evolution, Inheritance
Spice	Trade, Price equilibria
Cultural traits	Tribes, War, Migration

Our Approach–Background

- Students arrive at a boarding school having no friends and not knowing anyone.
- Each student occupies a position in Social Space.
- Students make friends at each time step according to a specified set of simple rules.

Student Social Space



- Social Space is a latent variable. It is a useful proxy for that which we cannot measure.
- Perhaps the axes are **IQ** and **Social Status**.
- Key idea: Students move towards their friends in Social Space.

Static Equation Model

The basic p-friendship model:

$$\log(\frac{p_{ij}}{1 - p_{ij}}) = \beta_0 + \beta_s\, s_i + \beta_r\, r_j + \delta\, X_{ij} - |z_i - z_j|$$

⋆ **p-Model:**

$$logit(p_{ij}) = \beta_0 + \delta_1 \left(\mathbf{I}(Sex_i = Sex_j) - \overline{\mathbf{I}(Sex_i = Sex_j)}\right) + \beta_s\,(s_i + s_j) - |z_i - z_j|$$

- $logit(p_{ij})$ is the *degree* of friendship between agents i and j. p_{ii} is undefined.
- β_0 is the baseline degree of friendship between any two agents.
- $\mathbf{I}(Sex_i = Sex_j)$ is an indicator whether agents i and j are of the same **Sex**. This dyadic covariate is centered about its mean to retain the interpretation of the baseline degree of friendship β_0.
- δ_1 is the sensitivity of friendships to same **Sex**.
- **Charisma** $s_i \sim \mathrm{N}(0, 1)$. Both the sender and the receiver have **Charisma** and both are equally weighted when making friendships.
- β_s is the sensitivity of friendships to **Charisma**.

Agent Model

⋆ Rules for Agent Model:

- Rule 0. Twenty agents start randomly at time=1 on a (20×20) grid in 2-dimensional Social Space.
- Rule 1. At every time step each agent i proffers a friendship to all agents $j \neq i$, and these proffers are accepted with probability p_{ij}.
- Rule 2. After new friendships are created, agents move a **"move.fraction"** towards the average of their friends' locations in Social Space.
- Rule 3. Agents are split evenly between the sexes. δ_1 is the sensitivity of friendships to same **Sex**.
- Rule 4. The **Charisma** (sociality s_i) of each agent is added to the model. $s_i \sim \mathrm{N}(0, 1)$. β_s is the sensitivity of friendships to **Charisma**.

Evaluation of Rules

How do changes in the set of rules change the results?

- Compare statistics of the network generated by different sets of rules
  - Average number of friends
  - Time until "perfect" clustering
  - Number of clusters
  - Number of completed triads
  - Number of opposite sex friends
  - Net distance moved by all agents in Social Space

Evaluation of Rules: Model 1

⋆ p-Model 1:

$$logit(p_{ij}) = \beta_0 - |z_i - z_j|$$

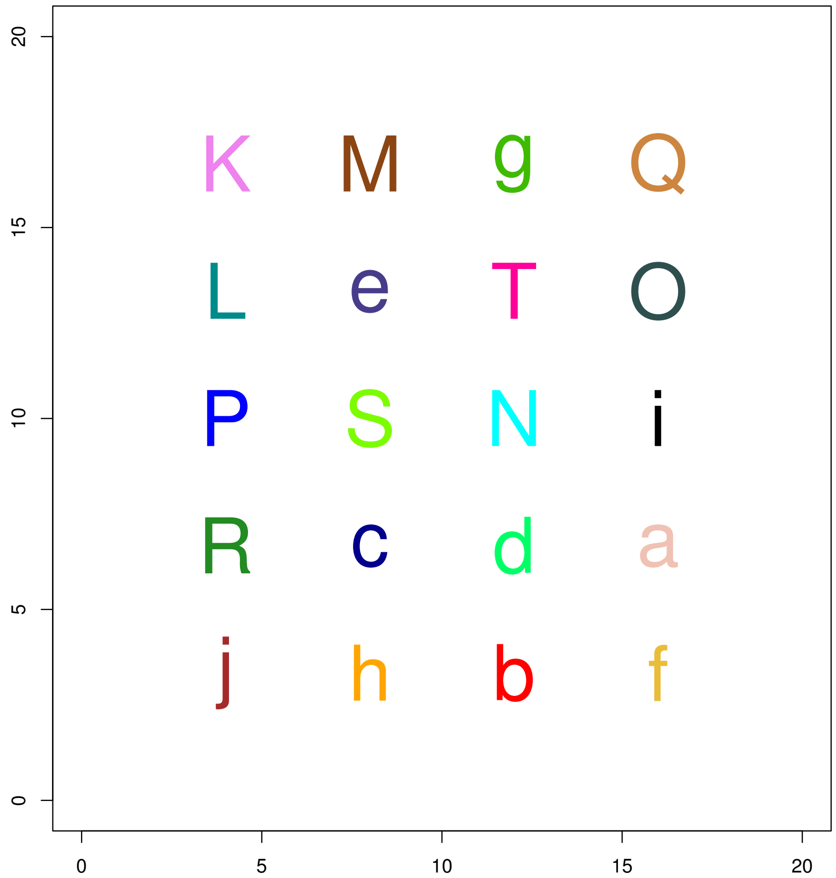
- $logit(p_{ij})$ is the *degree* of friendship between agents i and j.
- β_0 is the baseline degree of friendship between any two agents.
- **i** = 1, . . . , 20. **j** = 1,. . . , 20. The degree of friendship between an agent and itself, $logit(p_{ii})$, is undefined.
- z_i is the position of agent i in two-dimensional Social Space. $|z_i - z_j|$ is the distance between agents i and j.

⋆ Rules for Agent Model 1:

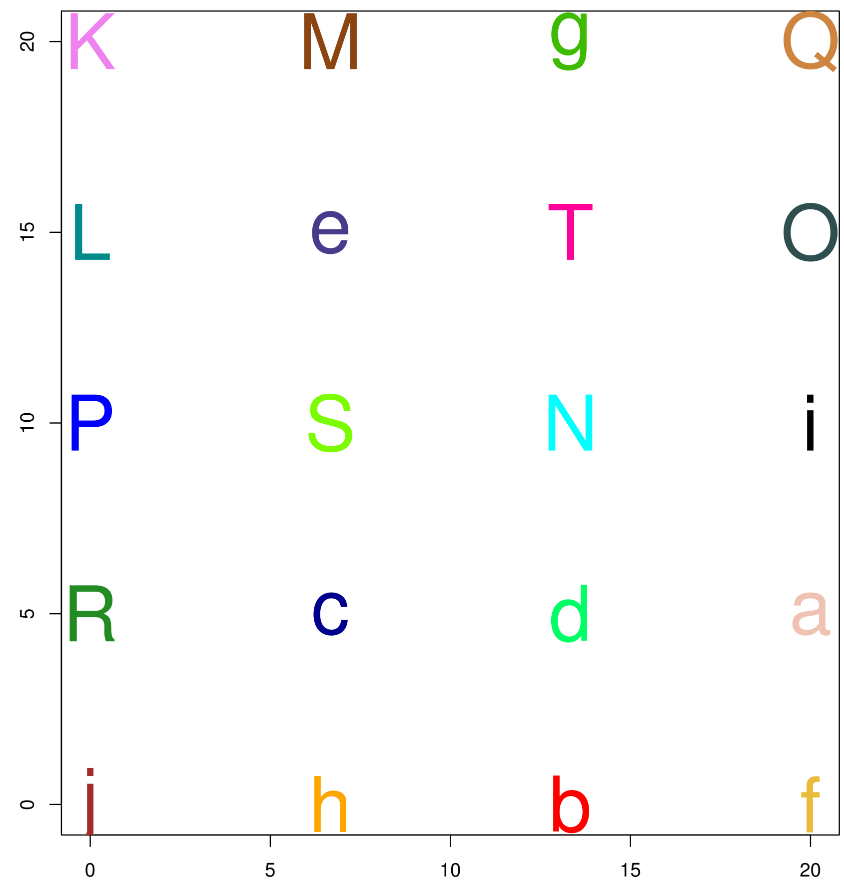
- Rule 0. Twenty agents start randomly at time=1 on a (20×20) grid in 2-dimensional Social Space.
- Rule 1. At every time step each agent i proffers a friendship to all agents $j \neq i$, and these proffers are accepted with probability p_{ij}.
- Rule 2. After new friendships are created, agents move a **"move.fraction"** towards the average of their friends' locations in Social Space.

Evaluation of Rules: Model 1 and 1b

Initial Locations in Social Space for Model 1



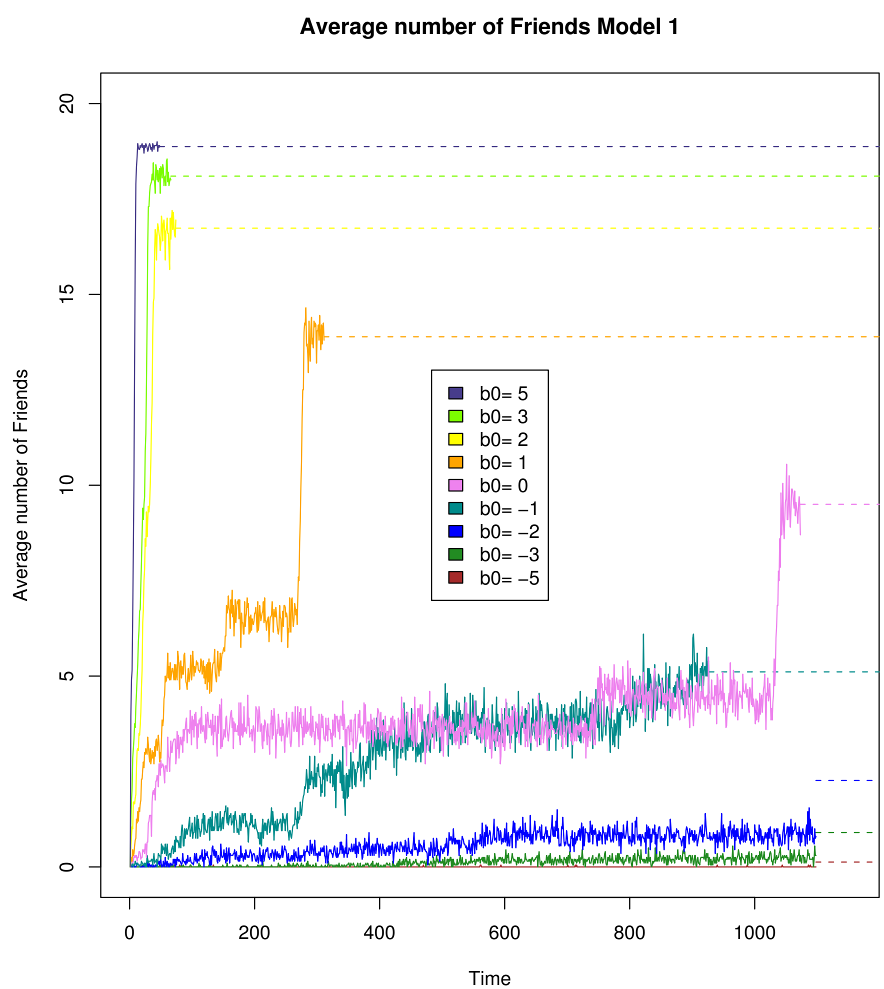
Initial Locations in Social Space for Model 1b

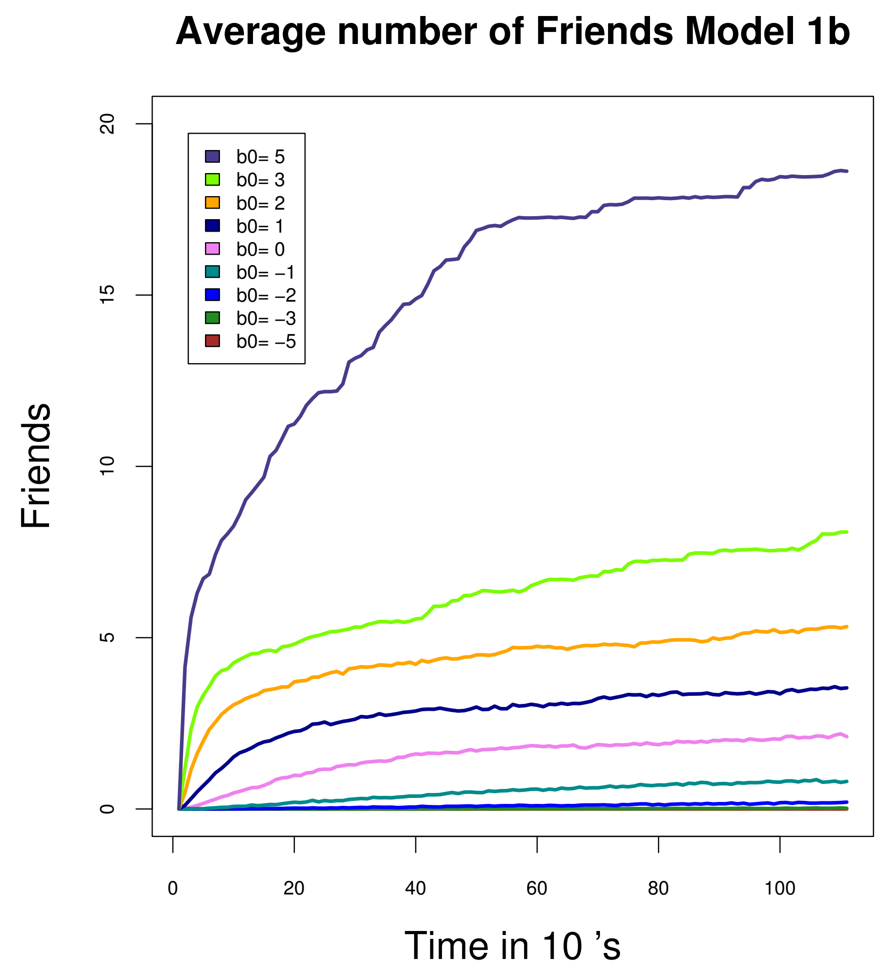


$$logit(p_{ij}) = \beta_0 - |z_i - z_j|$$

Rule 0. Size of Social Space

Evaluation of Rules: Model 1 and 1b





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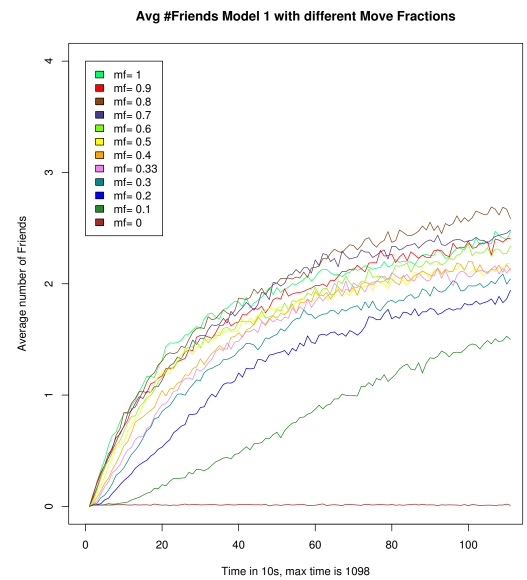
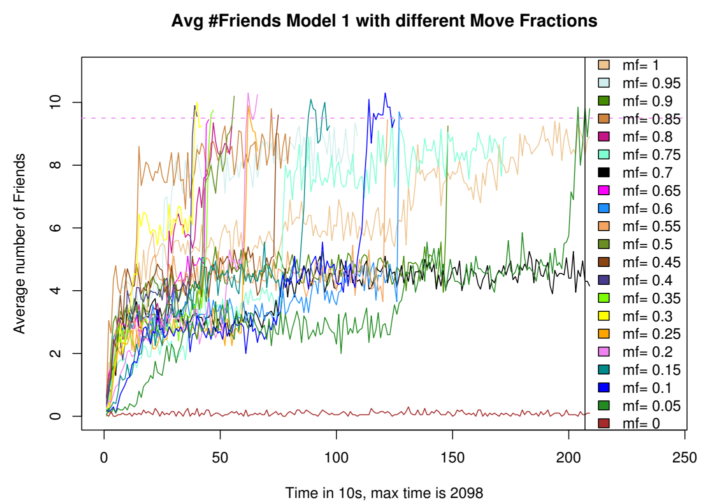
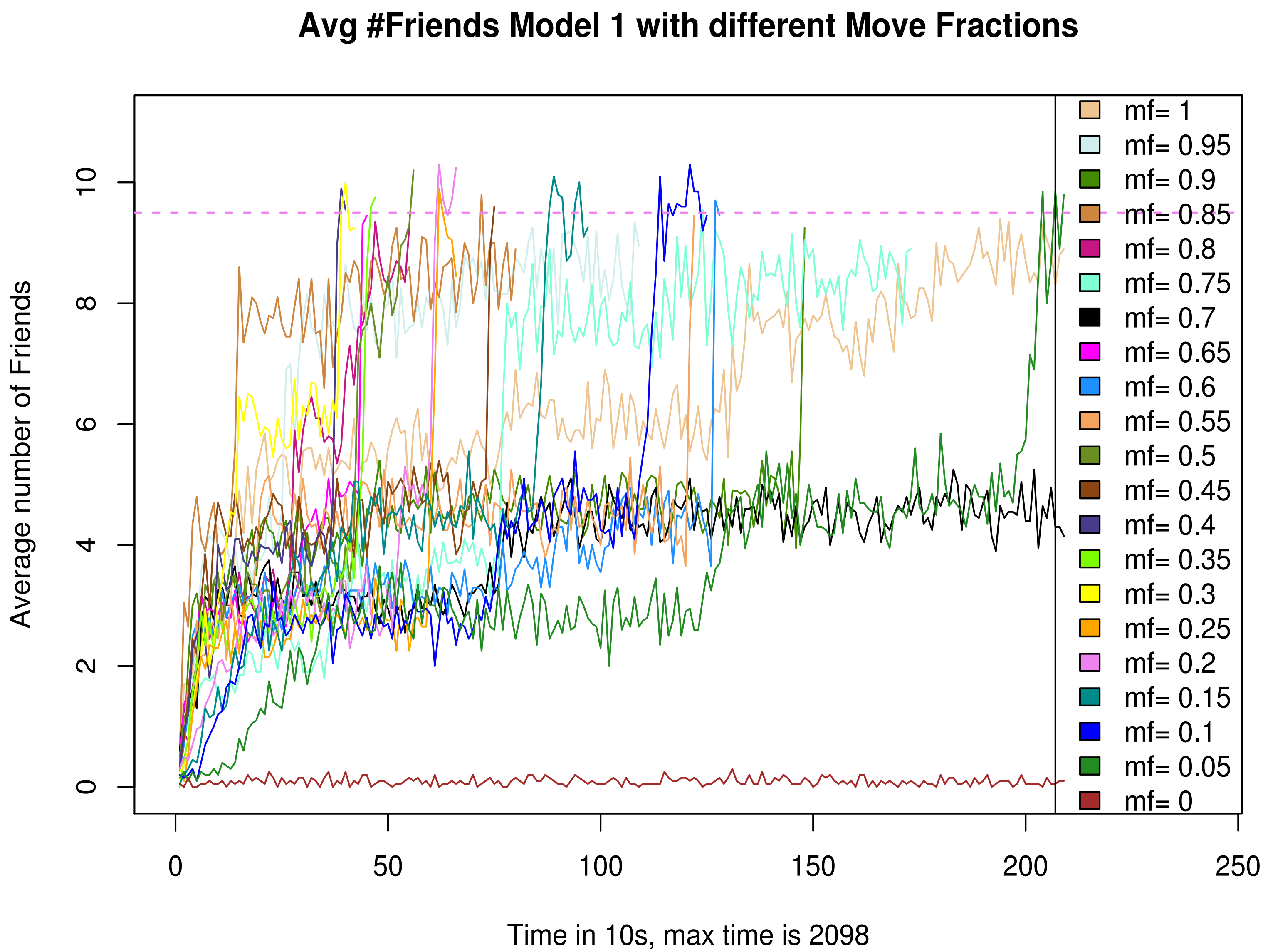




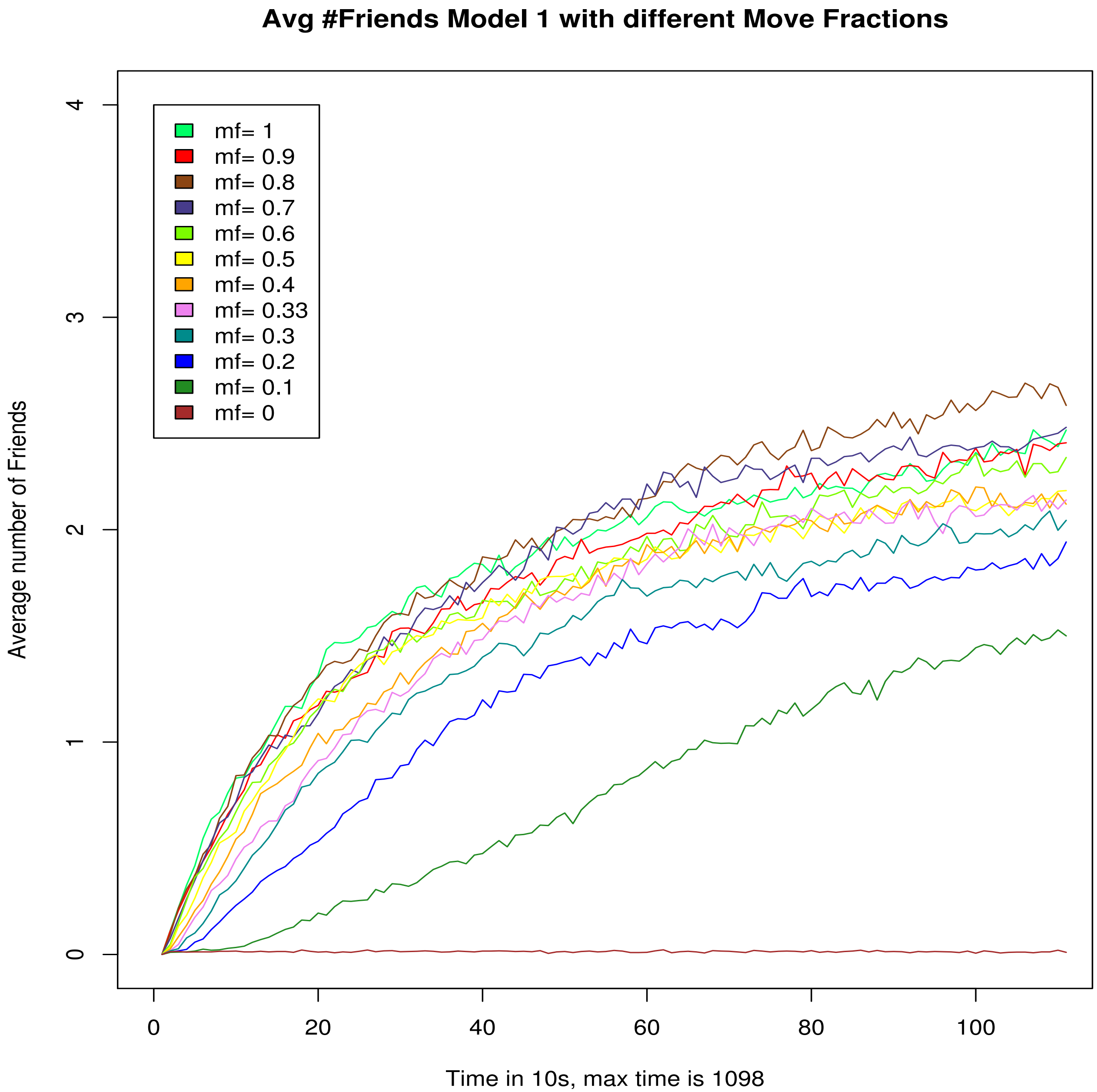
$$logit(p_{ij}) = \beta_0 - |z_i - z_j|$$

Rule 2. Students "move.fraction" towards friends in Social Space

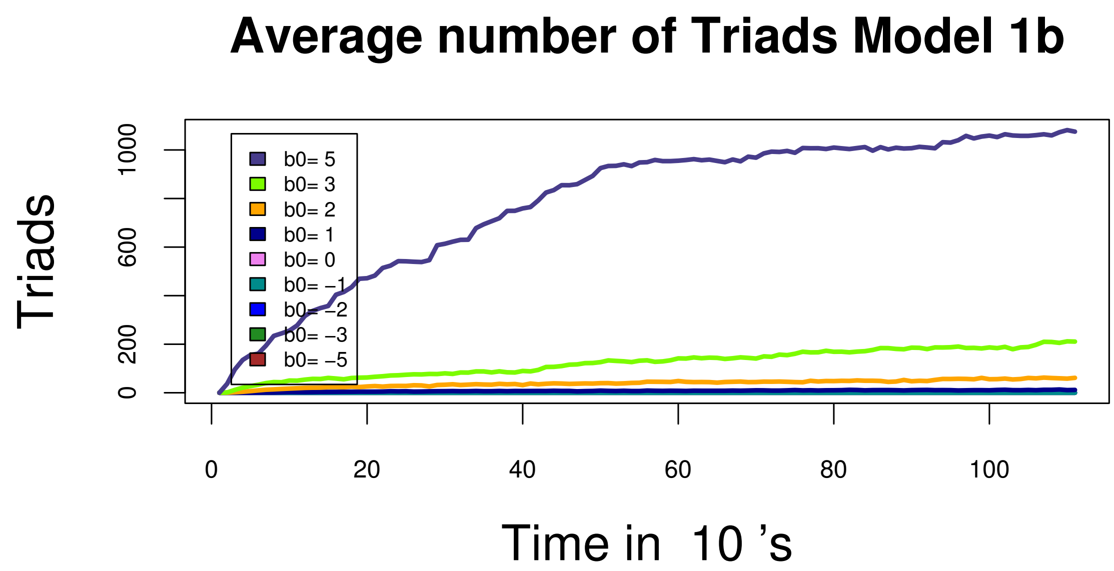
Evaluation of Rules: Model 1 and 1b

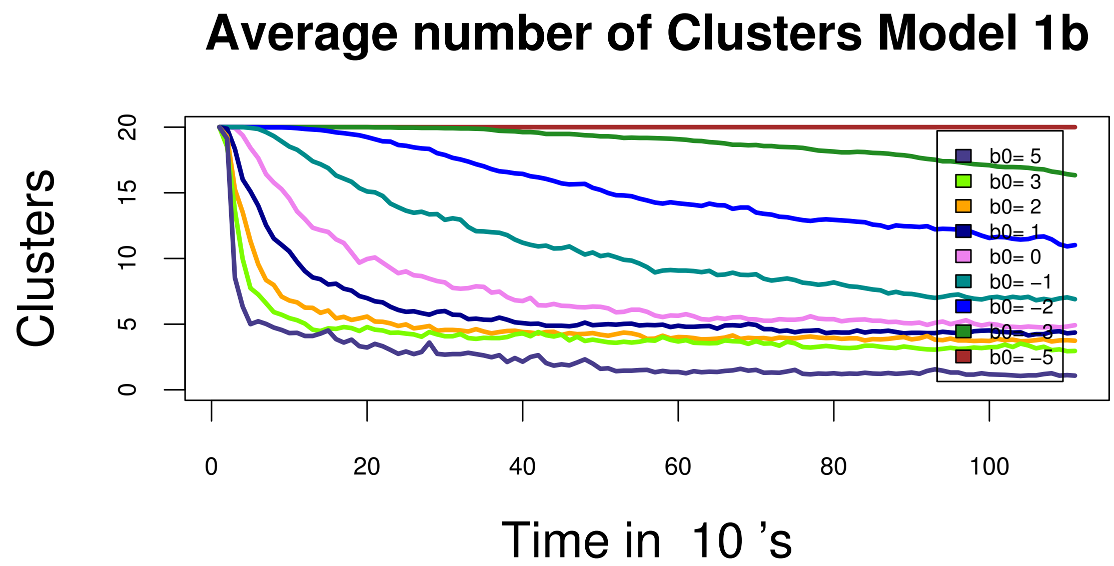


Evaluation of Rules: Model 1 and 1b



Evaluation of Rules: Model 1b





Gender is Added to Agent Model 2

⋆ p-Model 2:

$$logit(p_{ij}) = \beta_0 + \delta_1 \left(\mathbf{I}(Sex_i = Sex_j) - \overline{\mathbf{I}(Sex_i = Sex_j)}\right) - |z_i - z_j|$$

- β_0 is the baseline degree of friendship between any two agents.
- $\mathbf{I}(Sex_i = Sex_j)$ is an indicator whether agents i and j are of the same **Sex**. This dyadic covariate is centered about its mean to retain the interpretation of the baseline degree of friendship β_0.

⋆ Rules for Agent Model 2:

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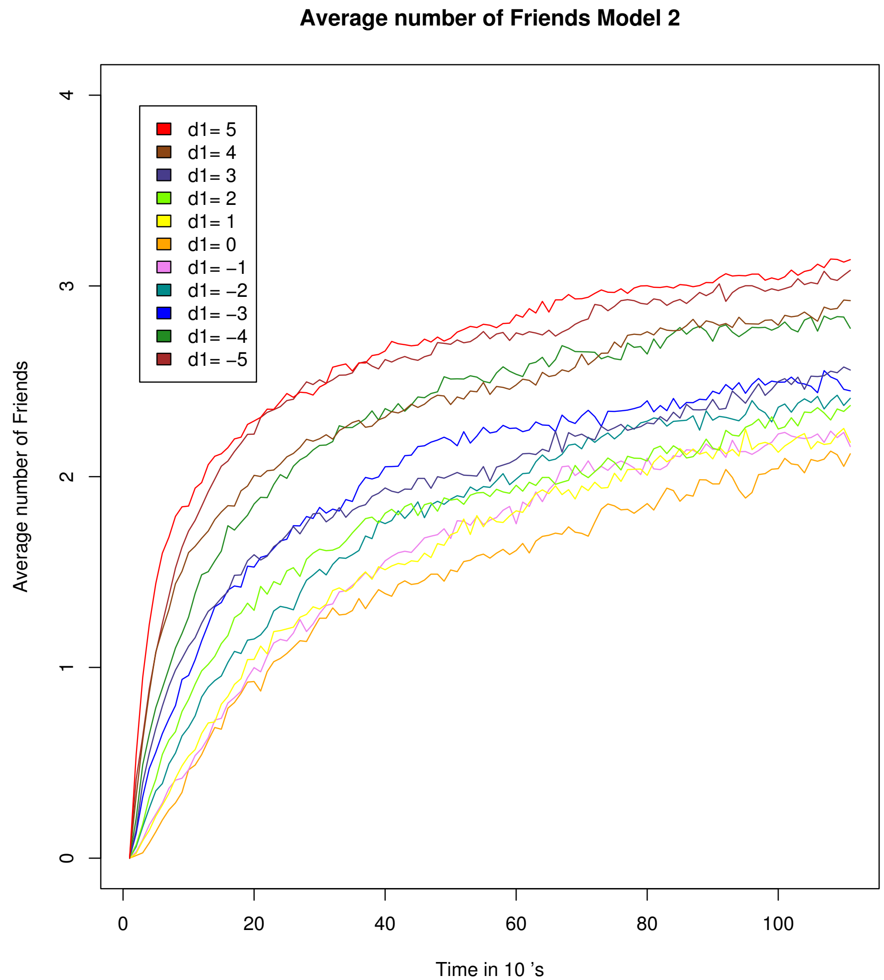
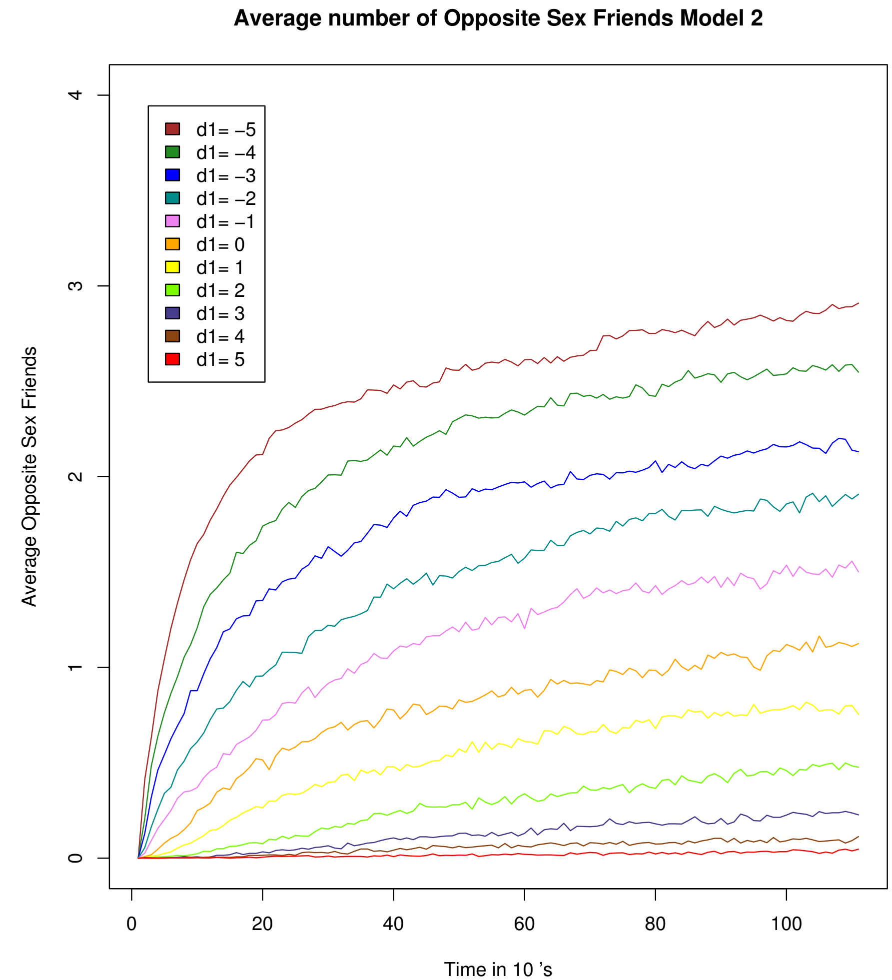
- Rule 1. At every time step each agent i proffers a friendship to all agents $j \neq i$, and these proffers are accepted with probability p_{ij}.
- Rule 2. After new friendships are created, agents move a "**move.fraction**" towards the average of their friends' locations in Social Space.
- Rule 3. Agents are split evenly between the sexes. δ_1 is the sensitivity of friendships to same **Sex**.

Evaluation of Rules: Model 2

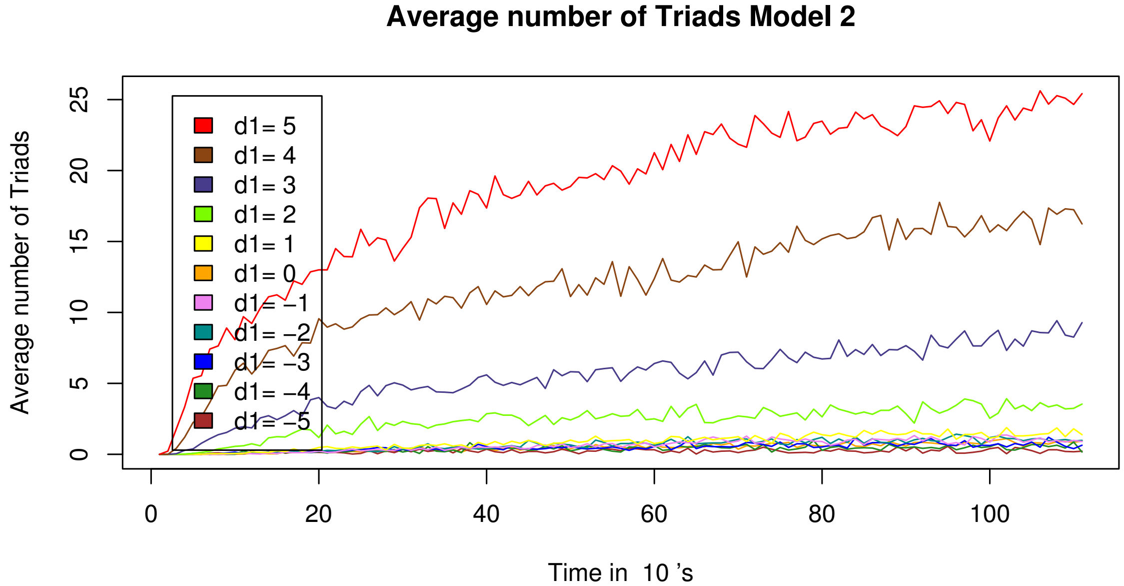


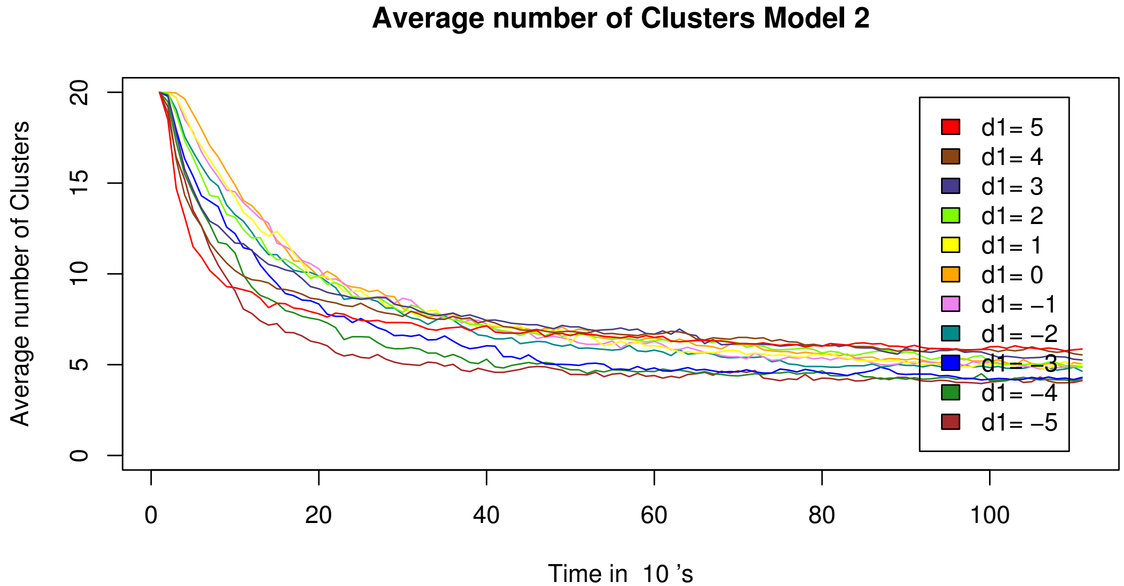


$$logit(p_{ij}) = 0 + \delta_1 \left(\mathbf{I}(Sex_i = Sex_j) - \overline{\mathbf{I}(Sex_i = Sex_j)} \right) - |z_i - z_j|$$

Rule 3. δ_1 **is the sensitivity of friendships to same Sex.**

Evaluation of Rules: Model 2





Evaluation of Rules: Model 2

- As long as **move.fraction** > 0, all the agents will eventually move together to form one perfect cluster. But it could be practially impossible based on β_0, δ_1, and the size of Social Space.

- Often one agent (starting in a corner of Social Space) will fail to make friends early and become isolated (nearly forever).

- Even when δ_1 was large, clusters of opposite sexes emerged in Social Space.

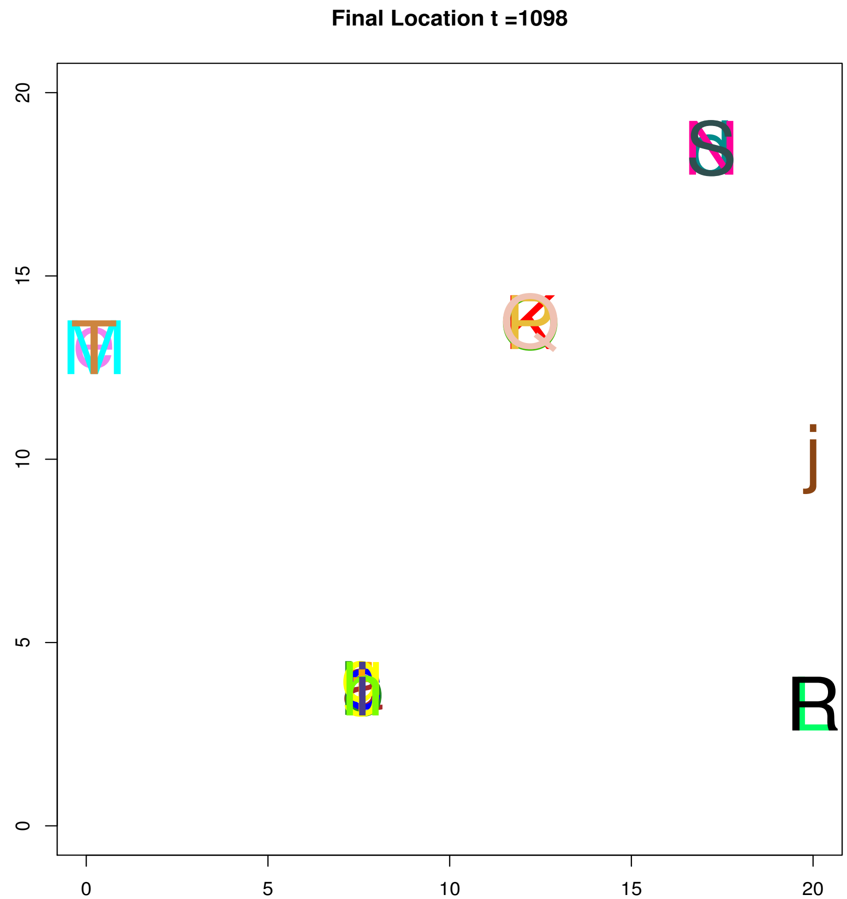
- Students in the same location in Social Space are not necessarily friends.

- In general, the larger δ_1 in absolute value, the higher the average number of friends.

- Adding sex into the equation drastically changes the dynamic behavior of the system. No longer do we observe one perfect cluster of agents at the end of the run. Sub-clusters in Social Space form and persist.

Agent Model 2 Example

- Social Space after 1098 iterations $\delta_1 = 2$ (preference for same sex):



- Six clusters of students emerge: A big cluster of 7 males, a cluster of 4 females, a smaller cluster of 2 females, two mixed clusters of 1 male and 2 females, and one lone male who never made any friends.

Charisma is Added to Agent Model 3

⋆ p-Model 3:

$$logit(p_{ij}) = \beta_0 + \delta_1\, Sex_{ij} + \beta_s\,(s_i + s_j) - |z_i - z_j|$$

- β_s is the sensitivity of friendships to **Charisma**.
- **Charisma** $s_i \sim \mathrm{N}(0,1)$. Both the sender and the receiver have **Charisma** and both are equally weighted when making friendships.

⋆ Rules for Agent Model 3:

- Rule 0. Twenty agents start randomly at time=1 on a (20×20) grid in 2-dimensional Social Space.
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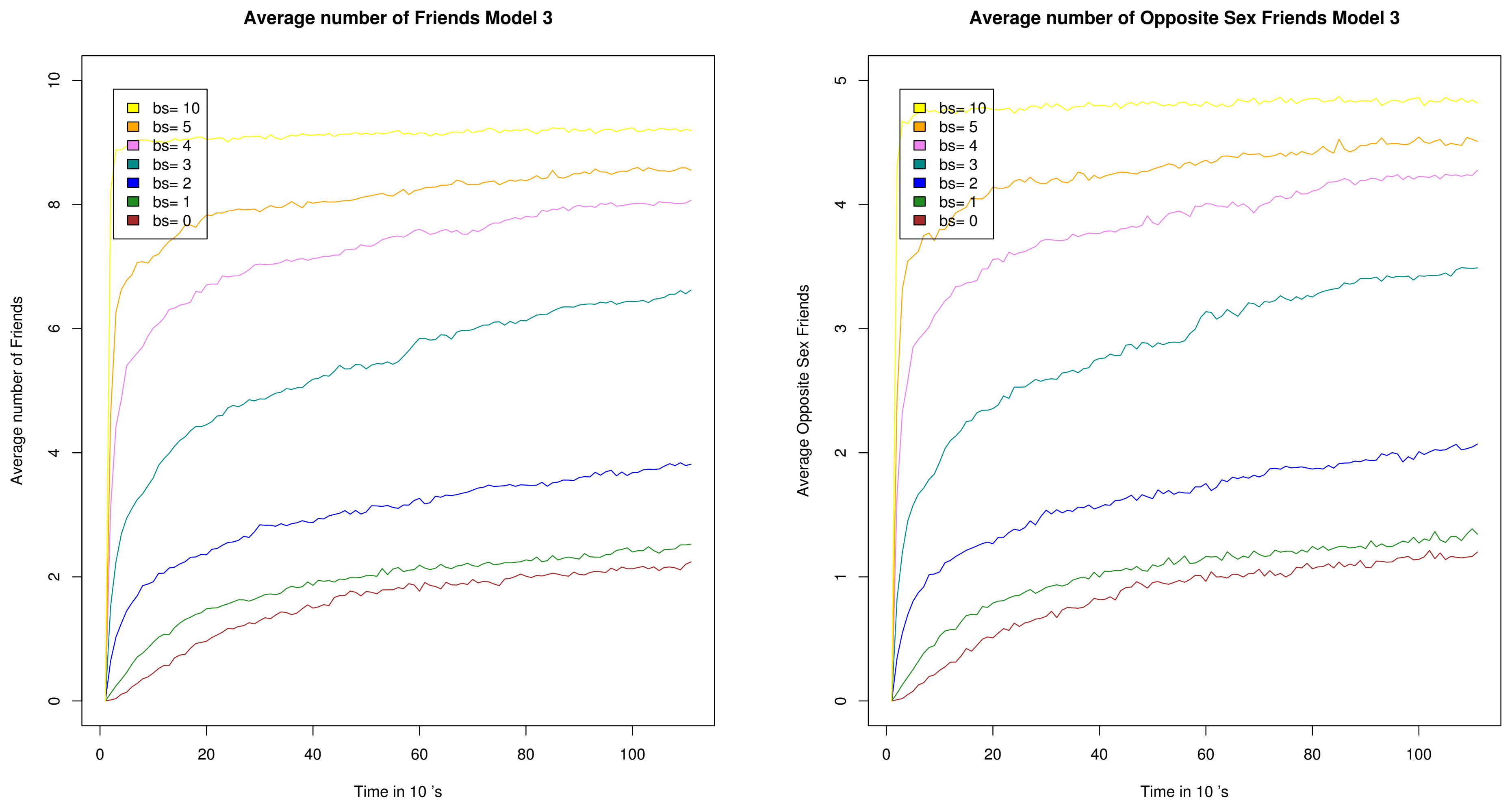
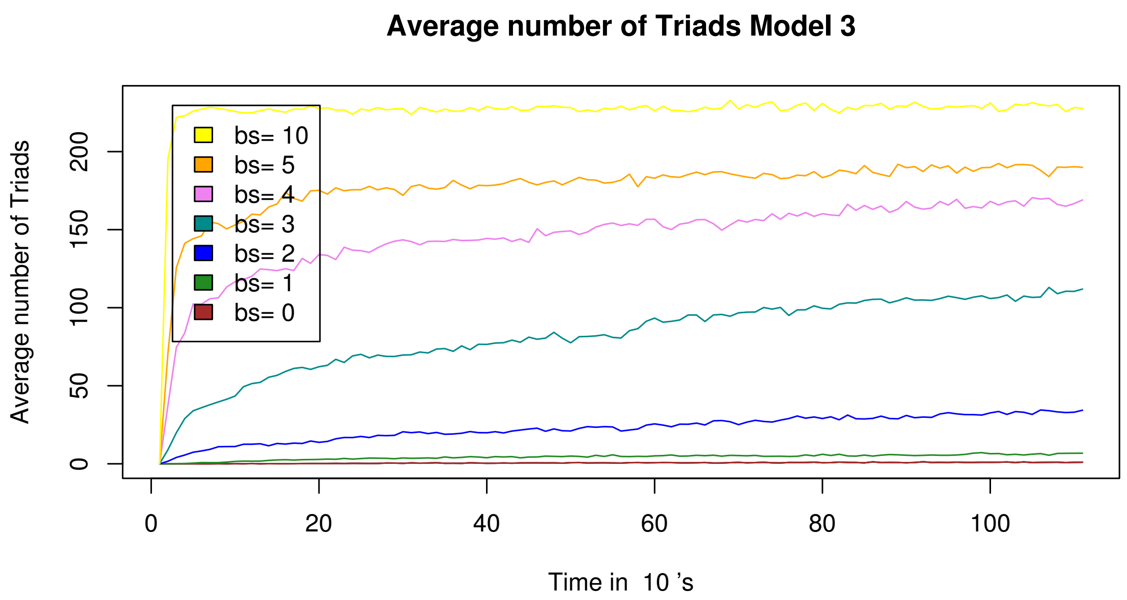
Evaluation of Rules: Model 3

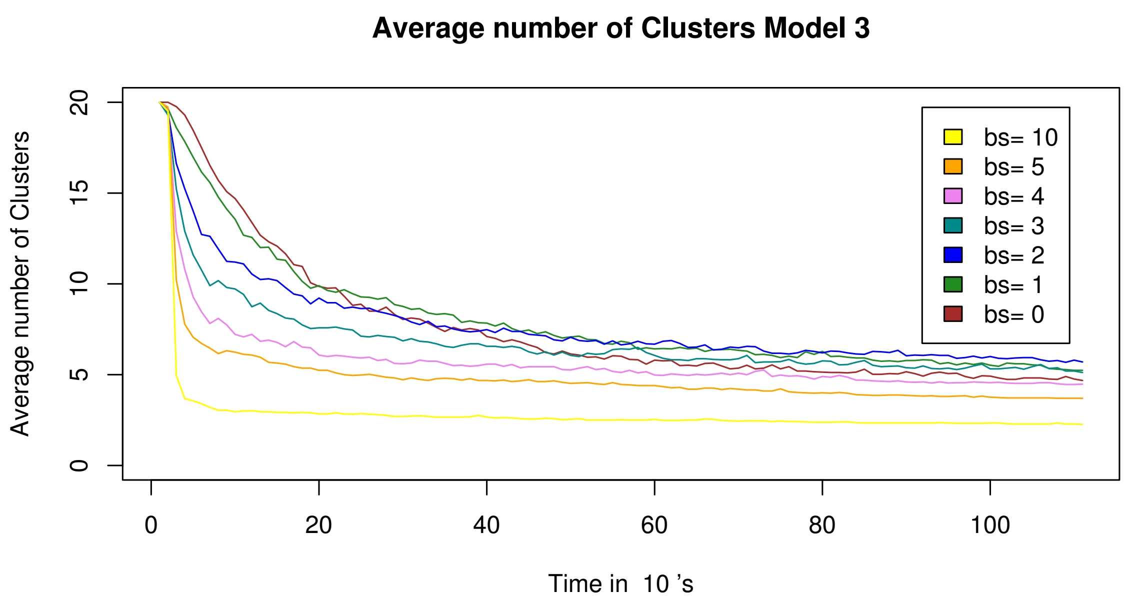


$$logit(p_{ij}) = 0 + \delta_1 \, Sex_{ij} + \beta_s \, (s_i + s_j) - |z_i - z_j|$$

Rule 4. β_s is the sensitivity of friendships to Charisma.

Evaluation of Rules: Model 3





Implications of Rules: Model 3

$$logit(p_{ij}) = \beta_0 + \delta_1\, Sex_{ij} + \beta_s\, s_i + \beta_r\, r_j - |z_i - z_j|$$

$$logit(p_{ij}) = 0 + \delta_1\, Sex_{ij} + \beta_s\, (s_i + s_j) - |z_i - z_j|$$

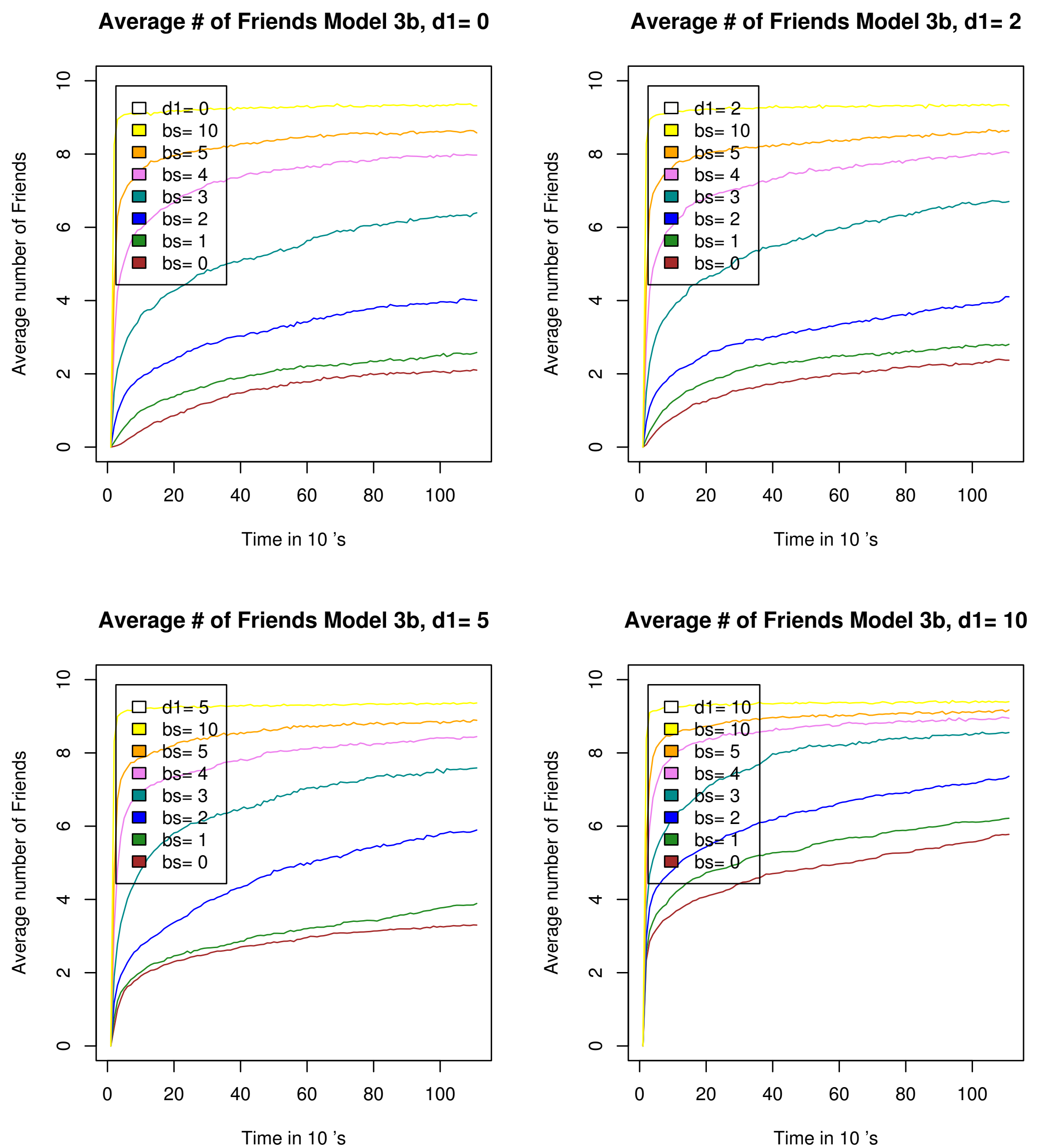
- **Charisma** of the sender s_i and the receiver $r_j = s_j$ are equally important in making friendships since the coefficients are equal ($\beta_s = \beta_r$) and the ties are undirected. What if there are differential sender and receiver effects?

- Students with high **Charisma** make lots of friends and live in large clusters. They might even bring everbody together into a perfect cluster.

- The distribution of **Charisma** might make a difference in the behavior of the model.

- What results when **Charisma** β_s and **Sex** δ_1 both vary?

Varying Charisma and Sex in Model 3



Varying Charisma and Sex in Model 3

$$logit(p_{ij}) = 0 + \delta_1\, Sex_{ij} + \beta_s\,(s_i + s_j) - |z_i - z_j|$$

- 1. Only when $\delta_1 = 10$ is there a noticeable difference in **Average Number of Friends** for different values of β_s.
- 2. For all values of δ_1 (sensitivity to **Sex**), Average friends increases with an increase in β_s.
- 3. Average number of **Opposite Sex Friends** seems very high for all values of δ_1 when $\beta_s = 10$. For most values of δ_1, opposite sex friends increases with an increase in β_s.
- 4. The average final location is not a perfect cluster for any values of the parameters. Social Space seems to be big enough to get stable sub-clusters. Agents start far enough apart to have a high probability of remaining apart.
- 5. The **Number of Triads** is the same for all values of δ_1 (except slightly when $\delta_1 = 10$). There is a clear monotonic relationship between number of triads and β_s (for a given δ_1).
- 6. The **Number of Clusters** is also largely unaffected by δ_1 and the relationship between clusters and β_s is fairly strong.

New Rules

- New rules could be imposed to add complexity to the model.
  - Introduce enmity or hatred between the students (a repulsive force)
  - Add jealousy to the model
  - Make movement in social space less likely as time goes on
- Model 4 bases movement in social space on all the *probabilities* of friendship p_{ij} rather than on 0/1 friends.
  - Given the random assignment of genders and charisma, the model becomes deterministic.
- Could change the interpretation of social space from a distance model to Peter Hoff's inner product social space, or could allow students to belong to specific "classes".

Statistical Challenges

- Sufficient statistics for the social network
- Which summary statistics to use for the network?
- How does one summarize a story?
- Evaluating models and rules without data