Agent-Based Methods for Dynamic Social Networks

Eric Vance

David Banks

Institute of Statistics & Decision Sciences

Duke University

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Agent-Based Models

- Impose a few simple rules on agents, then study the aggregate effects of the resulting interactions.

- Complex social phenomena can be generated by individual agents acting according to the simple rules.

- **Evaluate each new rule.**

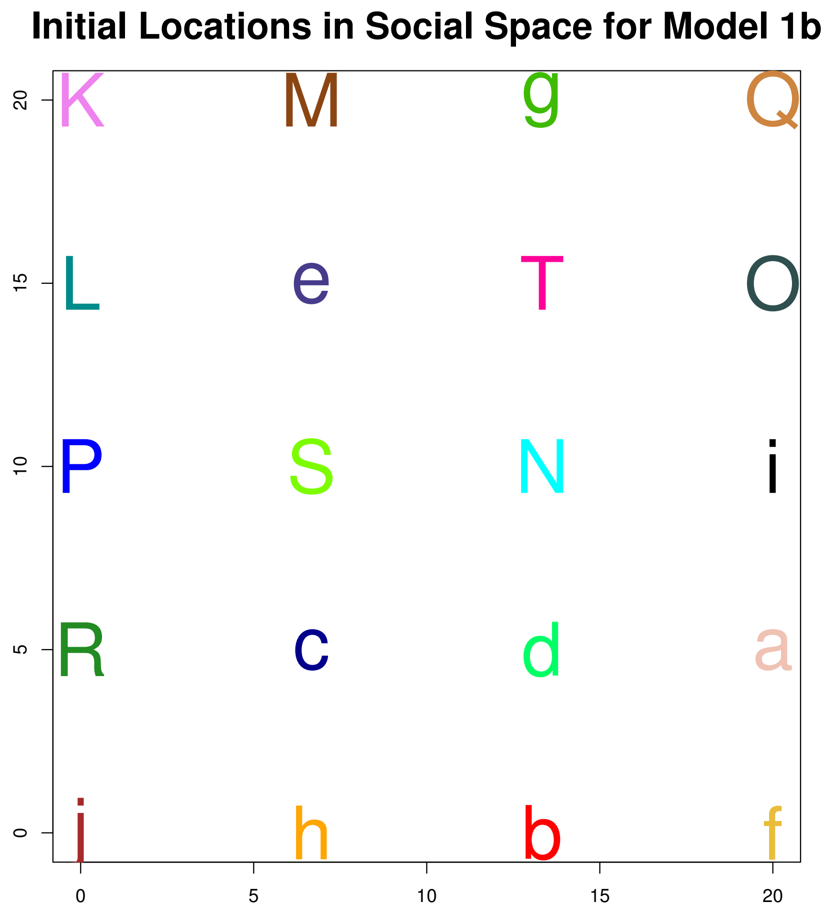
Hoff, Raftery, and Handcock Social Network Model

- Models the probabilities of friendships (p_{ij}) between actors
- $\text{logit}(p_{ij}) = \beta_0 + s_i + r_j + \beta_d X_{ij} - |z_i - z_j|$
  - Common intercept β_0, a baseline probability
  - Sender s_i random effect
  - Receiver r_j random effect
  - Vector of dyad-specific (observable) covariates X_{ij}
- Positions (z_i) in Social Space are latent variables.
- The distance between z_i and z_j in Social Space affects the probability of a friendship from $i \longrightarrow j$.
  – **Actors close together in social space are more likely to be friends.**

Our Approach–Background

- Students arrive at a boarding school having no friends and not knowing anyone.
- Each student occupies a position in Social Space.
- **Students make friends at each time step according to a specified set of simple rules which mimic the Hoff model.**

Student Social Space



- Social Space is a useful proxy for that which we cannot measure.
- Key idea: Students move towards their friends in Social Space.
- Students change their habits and interests to be more similar to their friends'.

Simulation of Model 1

http://www.stat.duke.edu/∼ervance/Sims/loc.anim.model1a.5-1.gif

http://www.stat.duke.edu/∼ervance/Sims/loc.anim.model1b.5-1.gif

Basic Model 1

⋆ p-Model 1:

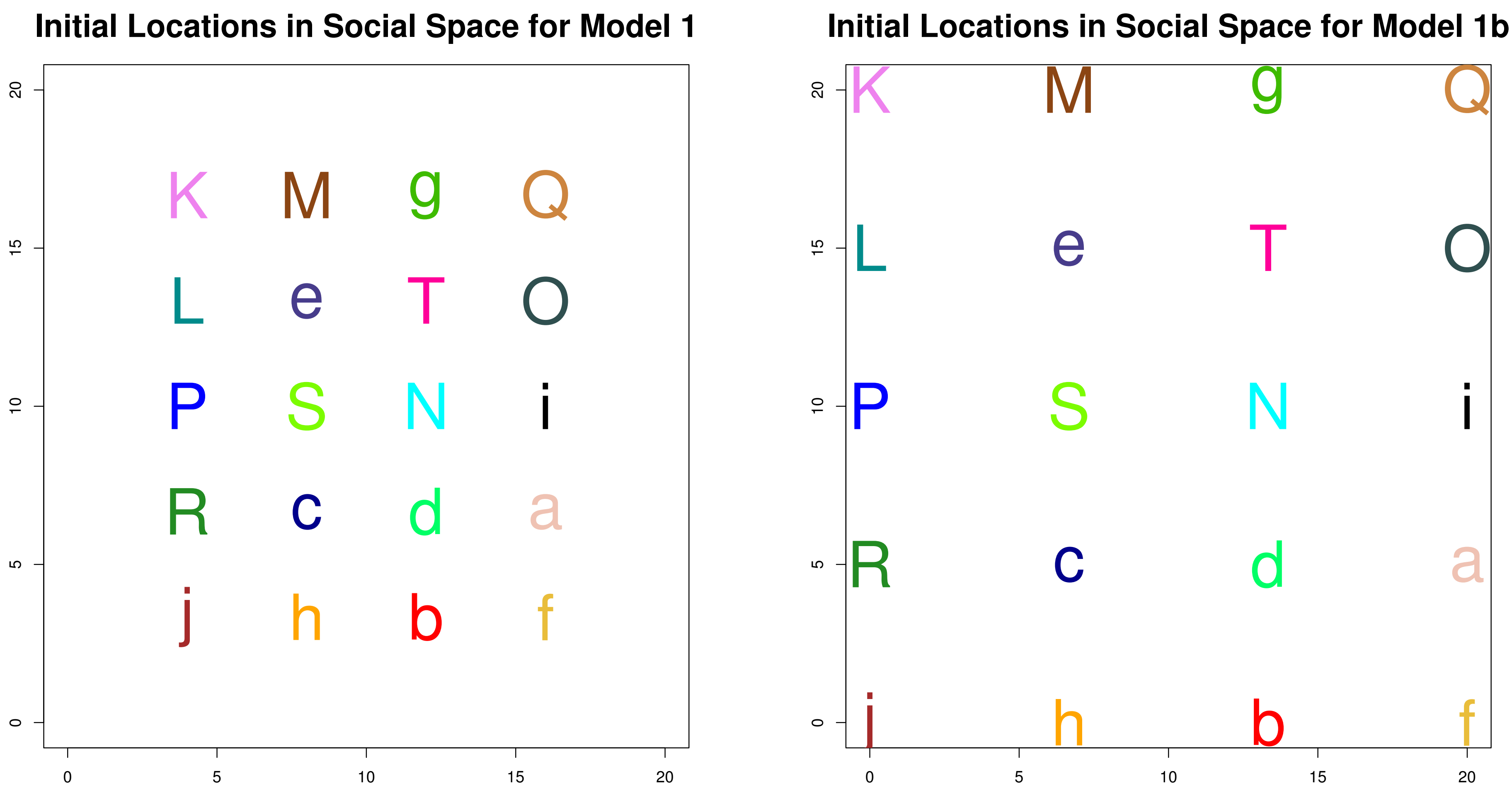
$$logit(p_{ij}) = \beta_0 - |z_i - z_j|$$

- $logit(p_{ij})$ is the *degree* of friendship between agents i and j.
- β_0 is the baseline degree of friendship between any two agents.
- **i** $= 1, \ldots, 20$. **j** $= 1, \ldots, 20$. The degree of friendship between an agent and itself, $logit(p_{ii})$, is undefined.
- z_i is the position of agent i in two-dimensional Social Space. $|z_i - z_j|$ is the distance between agents i and j.

⋆ Rules for Agent Model 1:

- Rule 0. Twenty agents start randomly at time=1 on a (20×20) grid in 2-dimensional Social Space.
- Rule 1. At every time step each agent i proffers a friendship to all agents $j \neq i$, and these proffers are accepted with probability p_{ij}.
- Rule 2. After new friendships are created, agents move a "**move.fraction**" towards the average of their friends' locations in Social Space.

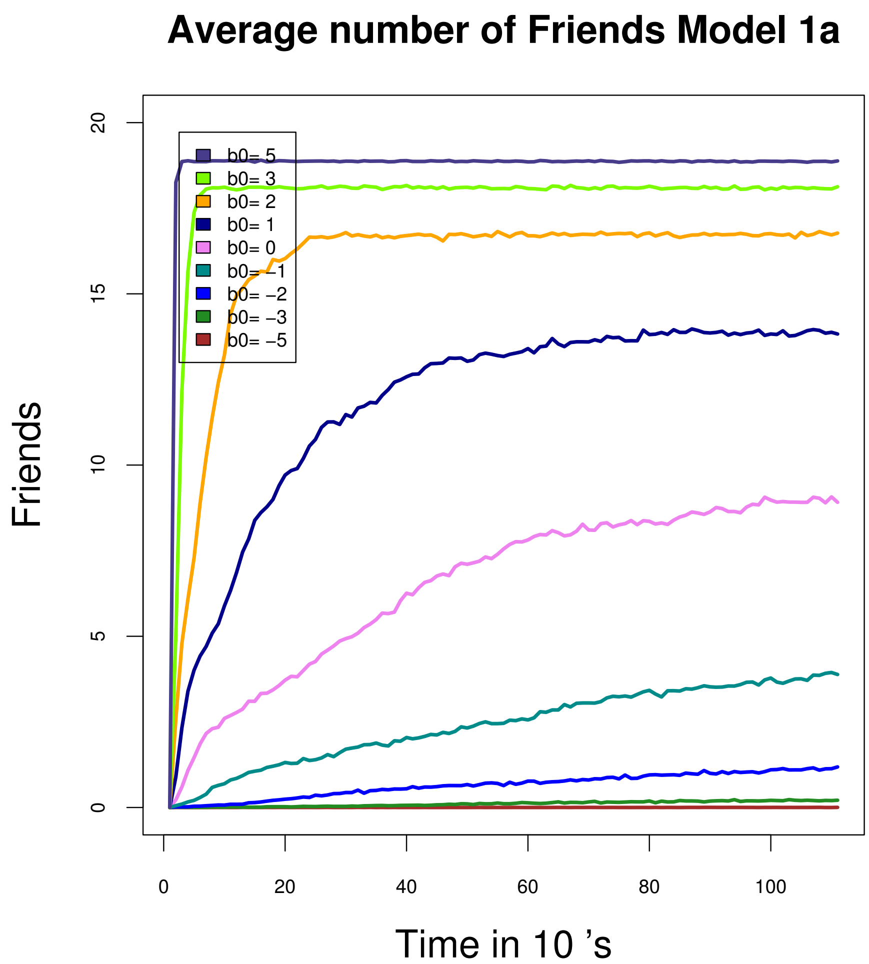
Rule 0: Model 1 and 1b

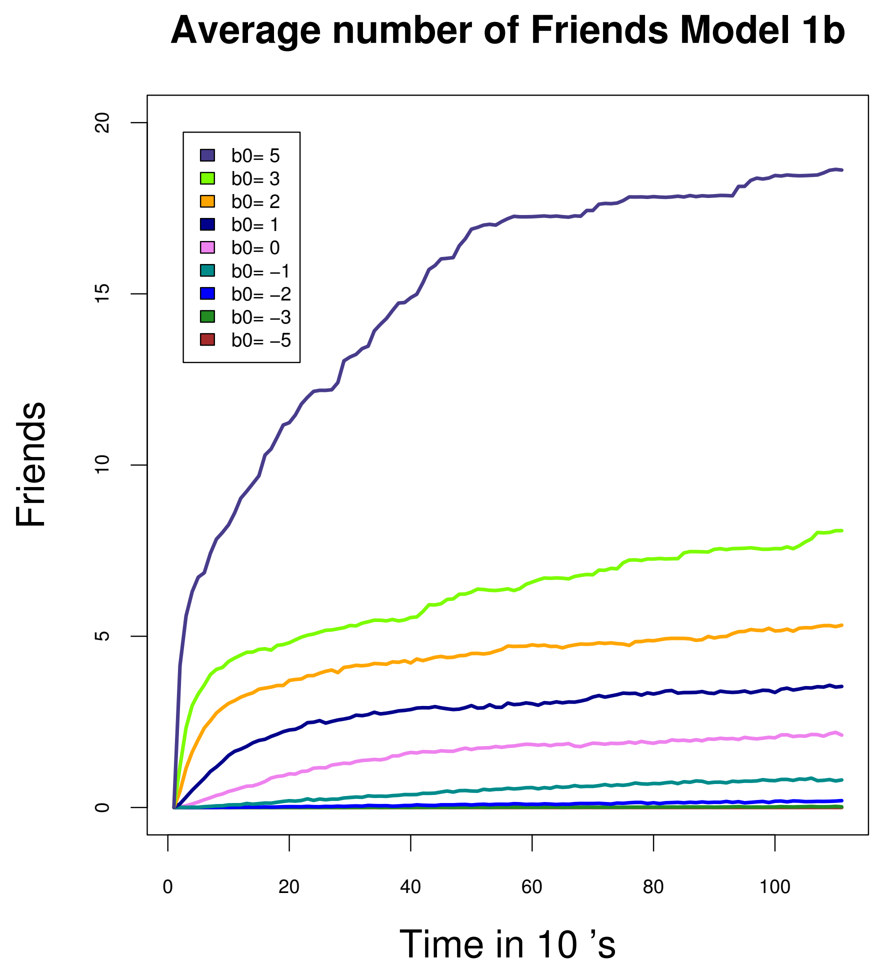


$$logit(p_{ij}) = \beta_0 - |z_i - z_j|$$

Rule 0. Size of Social Space

Evaluation of Rules 0 and 1





Rule 0. Size of Social Space

Rule 1. $logit(p_{ij}) = \beta_0 - |z_i - z_j|$

Gender is Added to Agent Model 2

⋆ p-Model 2:

$$logit(p_{ij}) = \beta_0 + \delta_1 \left(\mathbf{I}(Sex_i = Sex_j) - \overline{\mathbf{I}(Sex_i = Sex_j)}\right) - |z_i - z_j|$$

- β_0 is the baseline degree of friendship between any two agents.
- $\mathbf{I}(Sex_i = Sex_j)$ is an indicator whether agents i and j are of the same **Sex**. This dyadic covariate is centered about its mean to retain the interpretation of the baseline degree of friendship β_0.

⋆ Rules for Agent Model 2:

- Rule 0. Twenty agents start randomly at time=1 on a (20×20) grid in 2-dimensional Social Space.
- Rule 1. At every time step each agent i proffers a friendship to all agents $j \neq i$, and these proffers are accepted with probability p_{ij}.
- Rule 2. After new friendships are created, agents move a "**move.fraction**" towards the average of their friends' locations in Social Space.
- Rule 3. Agents are split evenly between the sexes. δ_1 is the sensitivity of friendships to same **Sex**.

Simulation of Model 2

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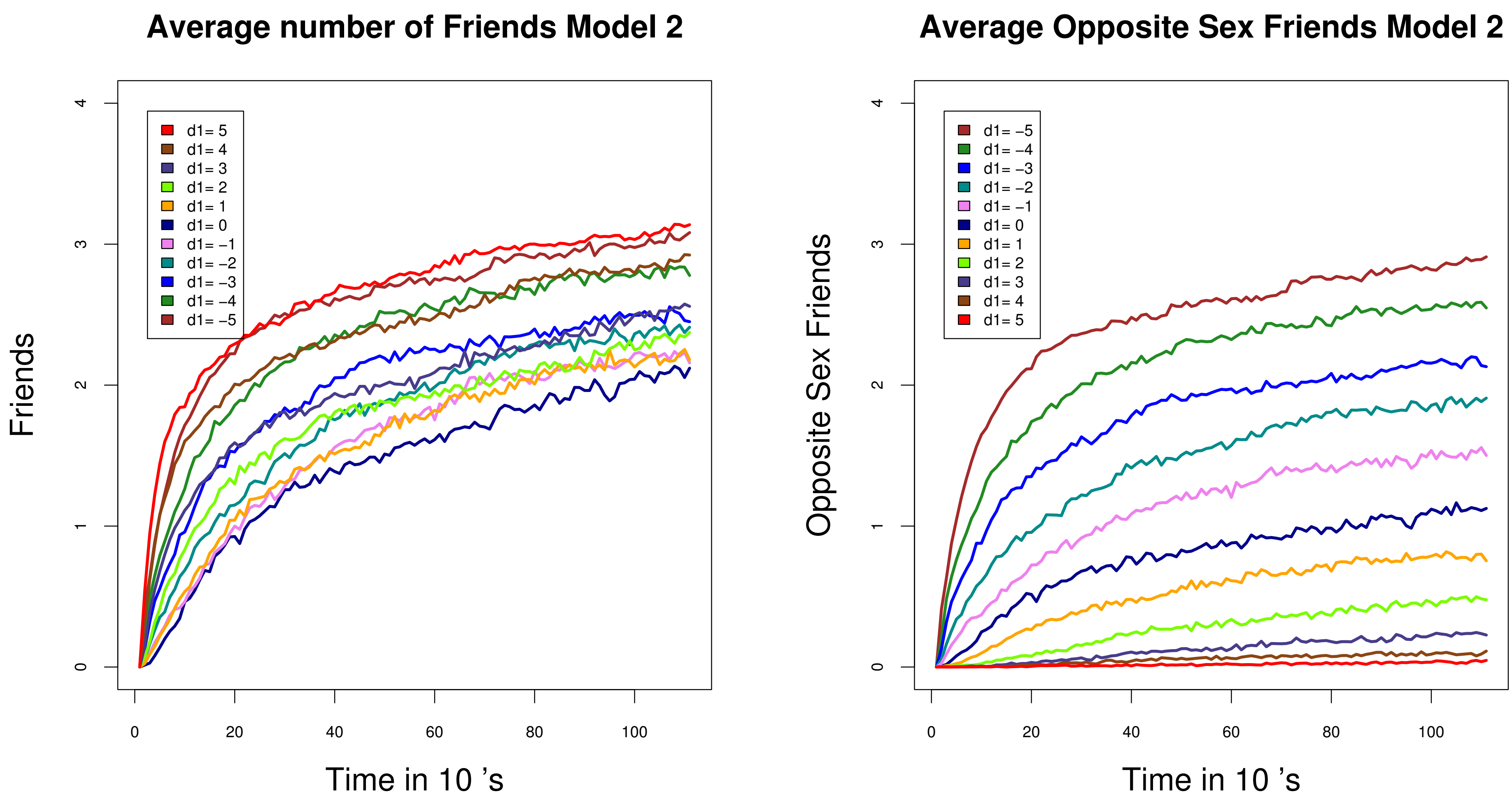
Evaluation of Rules: Model 2

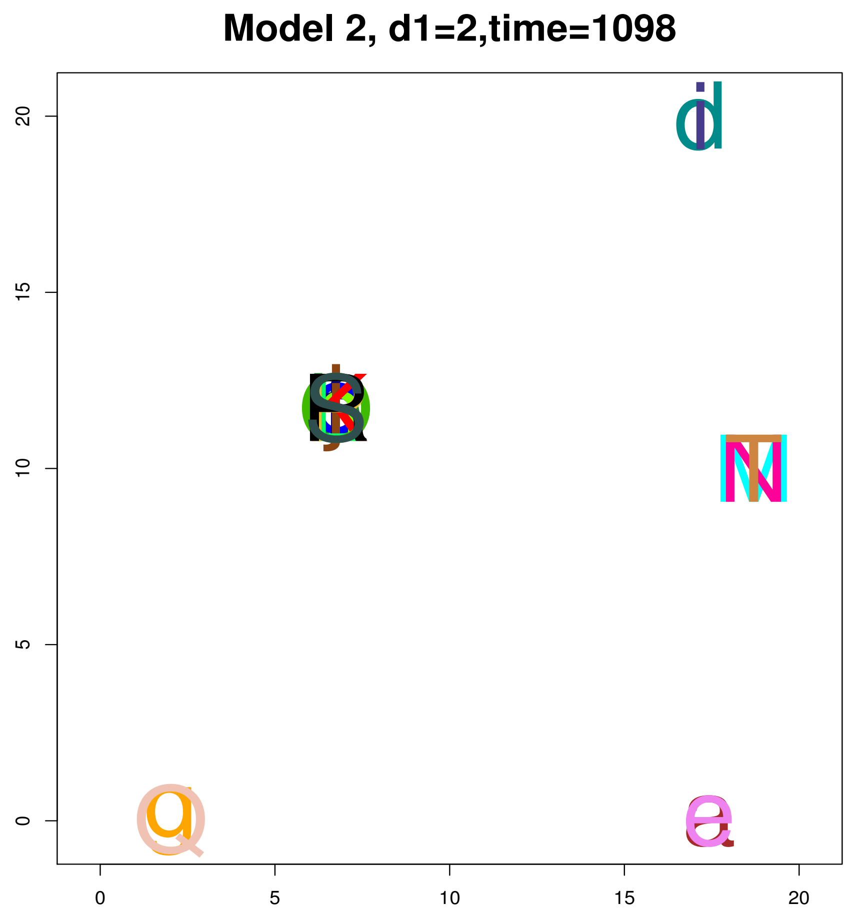


$$logit(p_{ij}) = 0 + \delta_1 \left(\mathbf{I}(Sex_i = Sex_j) - \overline{\mathbf{I}(Sex_i = Sex_j)} \right) - |z_i - z_j|$$

Rule 3. δ_1 **is the sensitivity of friendships to same Sex.**

Agent Model 2 Example

- Social Space after 1098 iterations $\delta_1 = 2$ (preference for same sex):



- Five clusters of students emerge: A big mixed cluster of 6 males and 5 females, a cluster of 3 males, two clusters of 2 females, and one male-female couple.

New Rules

- Model 3 assigns each student a "charisma" which makes them more or less attractive to potential friends.

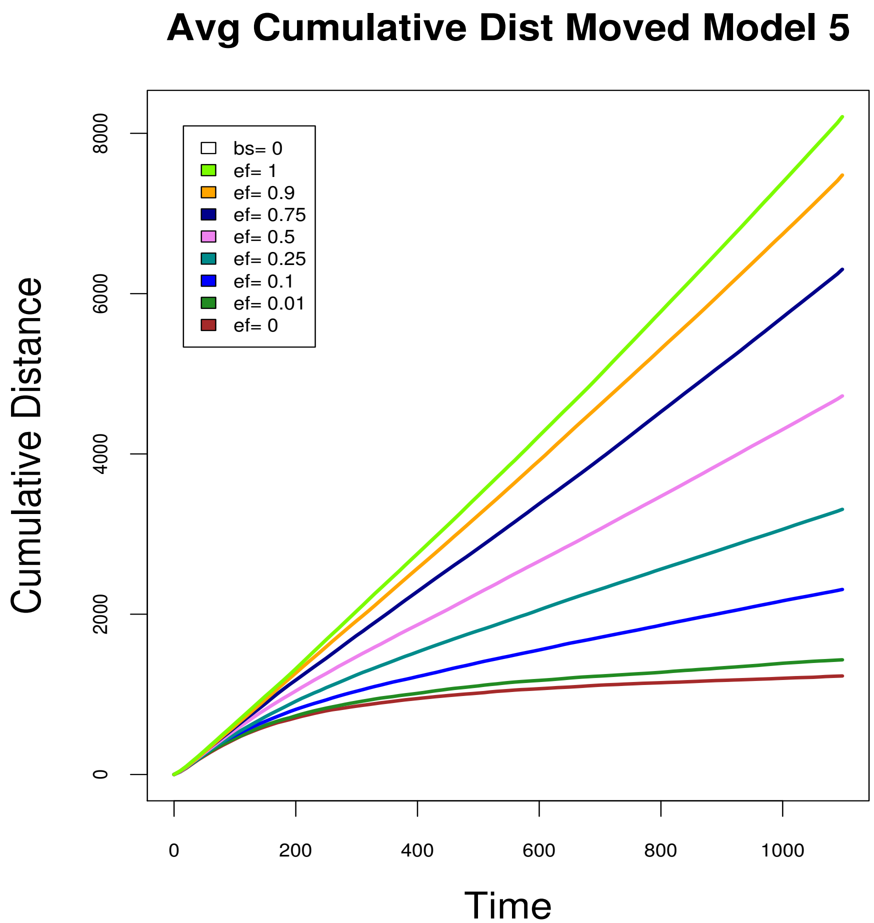
- Model 4 bases movement in social space on all the *probabilities* of friendship p_{ij} rather than on 0/1 friends.

- Model 5 adds a rule for enemies to the model. Students move away from their "enemies" in Social Space.

- New rules could be imposed to add further complexity.
  - Add history to the model. Agents remember previous friends and enemies.
  - Add jealousy to the model
  - Make movement in social space less likely as time goes on

Friends and Enemies in Model 5

- Students form enmity ties and move away from their enemies in Social Space.



- The higher the probability of making an enemy, the more the students move around in Social Space.

Simulation of Model 5

http://www.stat.duke.edu/∼ervance/Sims/loc.anim.model5-ef1.gif

Statistical Challenges

- Sufficient statistics for the social network
- Which summary statistics to use for the network?
- How does one summarize a story?
- Evaluating models and rules without data

Evaluation of Rules: Model 2

- As long as **move.fraction** > 0, all the agents will eventually move together to form one perfect cluster. But it could be practially impossible based on β_0, δ_1, and the size of Social Space.

- Often one agent (starting in a corner of Social Space) will fail to make friends early and become isolated (nearly forever).

- Even when δ_1 was large, clusters of opposite sexes emerged in Social Space.

- Students in the same location in Social Space are not necessarily friends.

- In general, the larger δ_1 in absolute value, the higher the average number of friends.

- Adding sex into the equation drastically changes the dynamic behavior of the system. No longer do we observe one perfect cluster of agents at the end of the run. Sub-clusters in Social Space form and persist.