The <u>Correct</u> Proof for Intersection

Intersection: If a JPD is positive, for all disjoint W, X, Y, Z,
$I(X, W|Z \cup Y) \wedge I(X, Y|Z \cup W) \Rightarrow I(X, Y \cup W|Z)$.

Proof:

$$I(X, W|Z \cup Y) \Rightarrow \quad P(XYZ)P(WYZ) = P(WXYZ)P(YZ) \tag{EQ 1}$$

$$I(X, Y|Z \cup W) \Rightarrow \quad P(WXZ)P(WYZ) = P(WXYZ)P(WZ) \tag{EQ 2}$$

Divide equation (2) by equation (1)

$$\frac{P(WXZ)}{P(XYZ)} = \frac{P(WZ)}{P(YZ)} \tag{EQ 3}$$

Sum W out of both sides of (3).

$$\sum_W \frac{P(WXZ)}{P(XYZ)} = \sum_W \frac{P(WZ)}{P(YZ)} \tag{EQ 4}$$

$$\frac{P(XZ)}{P(XYZ)} = \frac{P(Z)}{P(YZ)} \tag{EQ 5}$$

Rearrange the factors of (5):

$$\frac{P(XZ)}{P(Z)} = \frac{P(XYZ)}{P(YZ)} \tag{EQ 6}$$

Definition of conditional probability:

$$P(X|Z) = P(X|YZ) \tag{EQ 7}$$

Divide both sides of (1) by $P(YZ)P(WYZ)$

$$\frac{P(XYZ)}{P(YZ)} = \frac{P(WXYZ)}{P(WYZ)} \tag{EQ 8}$$

Definition of conditional probability:

$$P(X|YZ) = P(X|WYZ) \tag{EQ 9}$$

Substitute (7) into (9):

$$P(X|Z) = P(X|WYZ) \tag{EQ 10}$$

(10) is the desired consequent:

$$P(X|Z) = P(X|WYZ) \Rightarrow I(X, W \cup Y|Z)$$