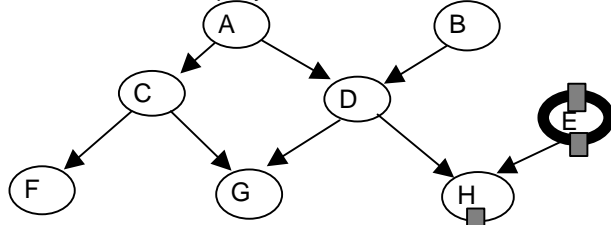


H/W 4 Solutions

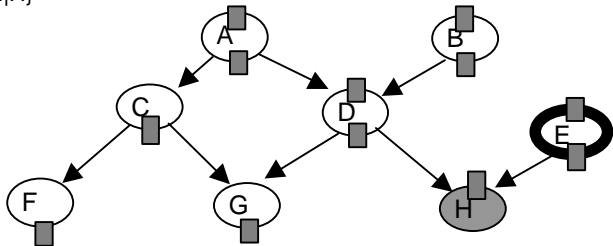
1. The first problem concerned the **Bayes Ball** algorithm.

a. $P(E)$ Bold circles are the query nodes. Shaded circles are the evidence.



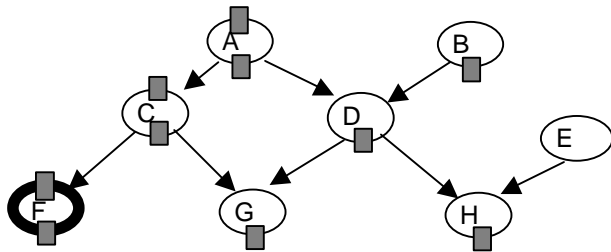
$N_p = \{E\}$
 $N_e = \{\}$

b. $P(E|H)$



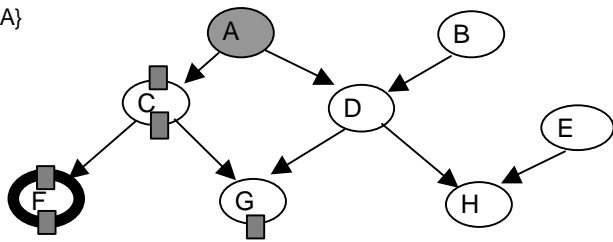
$N_p = \{A, B, D, E, H\}$
 $N_e = \{H\}$

c. $P(F)$



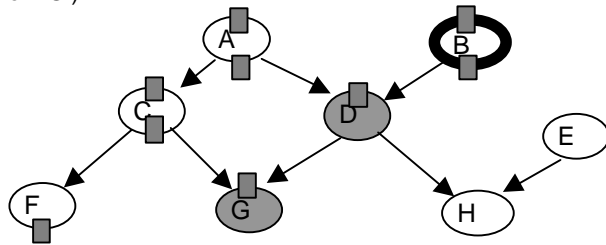
$N_p = \{F, C, A\}$
 $N_e = \{\}$

d. $P\{F|A\}$



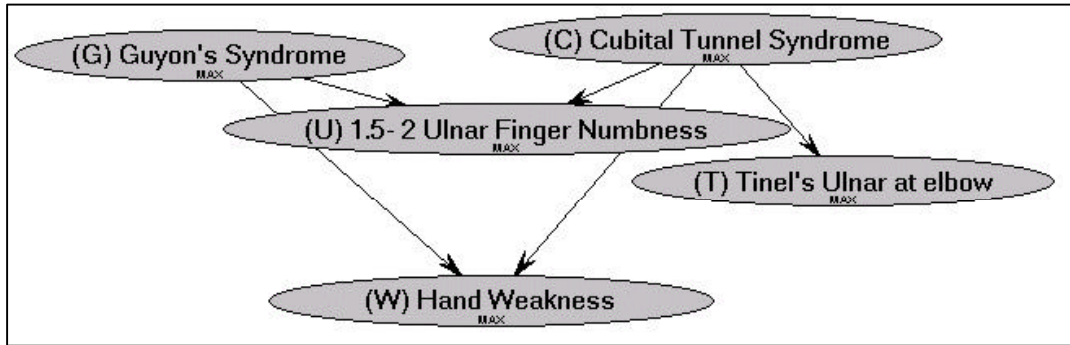
$N_p = \{F, C\}$
 $N_e = \{A\}$

e. $P\{B|D, G\}$ Almost everyone missed a part of this problem (the bounce back up to C from G.).



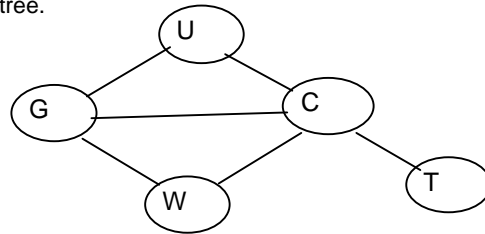
$N_p = \{B, D, A, C, G\}$
 $N_e = \{D, G\}$

2. Expert System Problem:



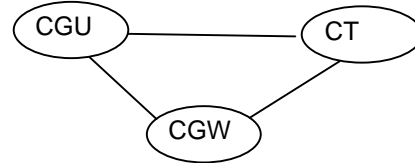
2.a construct junction tree.

(i) moralize

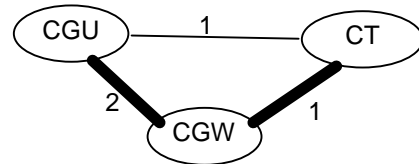


(ii) triangulate (done)

(iii) junction graph:



(iv) junction tree (Kruskal's algorithm)



(v) Assign potentials

P(C) to CGU

P(G) to CGU

P(W|CG) to CGW

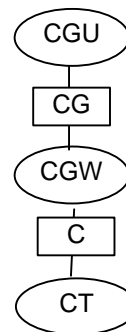
P(U|CG) to CGU

P(T|U) to CT

CGU	C=p		C=a		
	G=p	G=a	G=p	G=a	
⊃	0.000175	0.016625	0.001625	0.60515	a
	0.002975	0.049875	0.0273	0.006175	v
	0.01435	0.266	0.003575	0.006175	vd

CGW	C=p		C=a		
	G=p	G=a	G=p	G=a	
⊃	0.11	0.57	0.19	0.95	a
	0.89	0.43	0.81	0.05	p

CT	C=p		C=a		
	T		T		
	0.29	0.98			a
	0.71	0.02			p



The separators are all 1.0

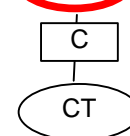
We don't actually have to propagate prior to using the junction tree.

2.b

CGU	C=p		C=a		
	G=p	G=a	G=p	G=a	
⊃	0	0	0	0	a
	0.002975	0.049875	0.0273	0.006175	v
	0	0	0	0	vd

CGW	C=p		C=a		
	G=p	G=a	G=p	G=a	
⊃	0	0	0	0	a
	0.89	0.43	0.81	0.05	p

CT	C=p		C=a		
	T		T		
	0.29	0.98			a
	0.71	0.02			p



Separator between CGU and CGW : $\Phi_S(CG)$

Separator between CGW and CT: $\Phi_S(C)$

Propagation:

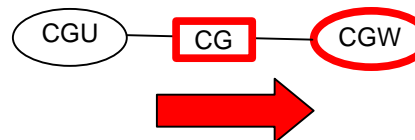
Absorption from CGU to CGW

$\Phi_{S^*}(CG) =$

	C=p		C=a		
	G=p	G=a	G=p	G=a	
	0.002975	0.049875	0.0273	0.006175	

$\Phi_{C^*}(CGW) =$

0.0026478	0.0214463	0.022113	0.00030875	W=p
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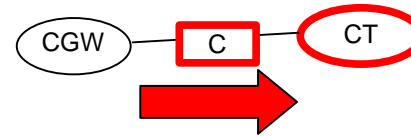
Rows that have a value of zero are suppressed

Absorption from CGW to CT

$$\text{Phi}_S^*(C) = \begin{array}{cc|c} C=p & C=a & \\ \hline & 0.024094 & 0.0224218 \end{array}$$

Phi_C*(CT) =

$$\text{CT} \begin{array}{cc|c} C=p & C=a & \\ \hline 0.0069873 & 0.0219733 & \\ 0.0171067 & 0.0004484 & \end{array}$$

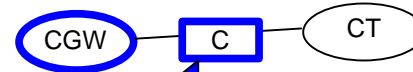


Absorption from CT to CGW

$$\text{Phi}_S^*(C) = \begin{array}{cc|c} & & \\ \hline & 0.024094 & 0.0224218 \end{array}$$

Phi_C*(CGW) =

$$\begin{array}{cc|cc} C=p & & C=a & \\ G=p & G=a & G=p & G=a \\ \hline 0.0026478 & 0.0214463 & 0.022113 & 0.00030875 \end{array} W=p$$



Rows that have a value of zero are suppressed

Absorption from CGW to CGU

$$\text{Phi}_S^*(C) = \begin{array}{cccc|c} & & & & \\ \hline & & & & \\ 0.0026478 & 0.0214463 & 0.022113 & 0.00030875 & \\ \text{Phi}_S^*(C)/\text{Phi}_S(C) = & 0.89 & 0.43 & 0.81 & 0.05 \end{array}$$

Phi_C*(CGU) =

$$\begin{array}{cccc|c} & & & & \\ \hline 0.0026478 & 0.0214463 & 0.022113 & 0.00030875 & U=v \end{array}$$



Rows that have a value of zero are suppressed

Note that the W=p row of CGU and the U=v row of CGU now have the same joint distribution over C and G.

Calculate $P(C|W=p, U=v)$ using the potential for CT. 0.5179751 This is $=(D157+D158)/SUM(D157:E158)$

Calculate $P(G|W=p, U=v)$ using the potential for CGU. 0.5323089 This is $=(D150+F150)/SUM(D150:G150)$

Calculate the joint distribution $P(CG|W=p, U=v)$ by normalizing $\text{Phi}_C(\text{CGU})$

$$\begin{array}{cc|cc} C=p & & C=a & \\ G=p & G=a & G=p & G=a \\ \hline 0.0569216 & 0.4610535 & 0.4753874 & 0.006637537 \end{array}$$

Note that C and G are anti-correlated.

2.c Start with the old junction tree (part 2.b)

Now observe that T = p

Multiply in evidence into $\text{Phi}_C(\text{CT})$

$$\text{CT} \begin{array}{cc|c} C=p & C=a & \\ \hline 0 & 0 & a \\ 0.0171067 & 0.0004484 & p \end{array}$$

Absorb evidence from CT to CGW and stop.

$$\text{Phi}_S^*(C) = \begin{matrix} & C=p & C=a \\ & 0.0171067 & 0.0004484 \end{matrix}$$

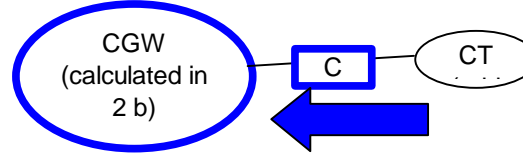
$$\text{Phi}_S^*(C)/\text{Phi}_S(C) = \begin{matrix} & C=p & C=a \\ & 0.71 & 0.02 \end{matrix}$$

$$\text{Phi}_C(\text{CGW}) = \begin{matrix} & C=p & G=a & C=a & G=p & G=a \\ \text{(old value)} & 0.0026478 & 0.0214463 & 0.022113 & 0.00030875 \end{matrix}$$

$$\text{Phi}_C^*(\text{CGW}) = \begin{matrix} & C=p & G=a & C=a & G=p & G=a \\ & 0.0018799 & 0.0152268 & 0.0004423 & 0.000006175 \end{matrix}$$

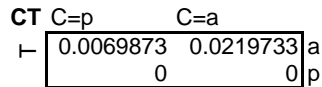
Calculate $P(C|W=p, U=v)$ using the potential for CGW. 0.9744557

Calculate $P(G|W=p, U=v)$ using the potential for CGW. 0.1322779



2.d

Start with the old junction tree (part 2.b)



Absorb evidence from CT to CGW and stop.

$$\text{Phi}_S^*(C) = \begin{matrix} & C=p & C=a \\ & 0.0069873 & 0.0219733 \end{matrix}$$

$$\text{Phi}_S^*(C)/\text{Phi}_S(C) = \begin{matrix} & C=p & C=a \\ & 0.29 & 0.98 \end{matrix}$$

$$\text{Phi}_C(\text{CGW}) = \begin{matrix} & C=p & G=a & C=a & G=p & G=a \\ \text{(old value)} & 0.0026478 & 0.0214463 & 0.022113 & 0.00030875 \end{matrix}$$

$$\text{Phi}_C^*(\text{CGW}) = \begin{matrix} & C=p & G=a & C=a & G=p & G=a \\ & 0.0007678 & 0.0062194 & 0.0216707 & 0.000302575 \end{matrix}$$

Calculate $P(C|W=p, U=v)$ using the potential for CGW. 0.241268 0.00698726 0.0289606 0.241268

Calculate $P(G|W=p, U=v)$ using the potential for CGW. 0.7747977

