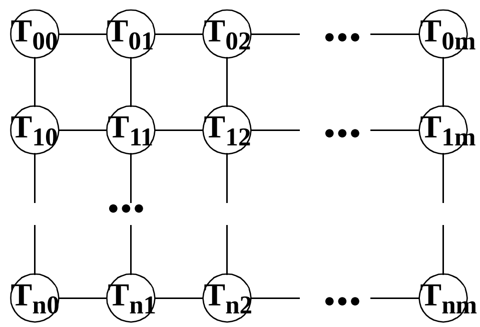
STA 294 Homework 2

Due 2/1/99

Cite work that you use outside of the assigned text and readings.

Oceanography (Levine)

1. (10) The undirected graph G below represents a model for the distribution of temperature measurements in the ocean. The model for this problem is a $n \times m$ rectangular lattice of continuous valued-variables. .



Find a decomposable graph G^D corresponding to G. Try to minimize the size of the largest clique. How many nodes are in the largest clique?

"Local" Independence in Directed Graphs

2. (10) What is the smallest set of nodes B in directed graph G that renders node x independent of the rest of the graph $X \setminus (B \cup \{x\})$. That is, what is the smallest B such that

$$I(\{x\}, X \setminus (B \cup \{x\}) | B)_G$$

Barren Nodes

3. (15) One of the objectives of probabilistic inference is to compute the posterior distribution over a set of variables (called *query variables,* Q) given values for *evidence or observation variables* (E). The distribution for this *query* is derived by marginalizing out all of the variables X that are neither query variables nor evidence variables. That is,

$P\{Q|E\} = \left(\sum_{X \setminus (Q \cup E)} P\{X\}\right) \Big/ \left(\sum_{Q \setminus E} P\{X\}\right) = K_E \left(\sum_{X \setminus (Q \cup E)} P\{X\}\right)$, where X is the set of all of the variables in DAG G and K_E is a constant of proportionality that depends only on the observed evidence E. A node x_b is *barren* with respect to the query $P\{Q|E\}$ if x_b has no children and is neither a query variable nor an evidence variable.

a. (8) Prove that a node that is barren for query $P\{Q|E\}$ can be removed from G without affecting the value of the query.

b. (7) Say that we incrementally remove nodes that are barren with respect to query $P\{Q|E\}$ until no node is barren. What nodes are left in G? Can we *prune* any more nodes from G?

Graphoids and Semi-Graphoids

4. (10) [CGH] Problem 5.4, parts (c) and (d).

5. (10) [CGH] Problem 6.2.

(P), (L), (G) and (F)

6. (30) This exercise discusses several algorithms for constructing an undirected graph given any joint probability distribution. For each algorithm, describe which Markov properties; (P), (L), (G), or (F); will necessarily hold for the constructed graph.

a. (10) U_1 constructs an undirected graph G by incrementally removing arcs from a complete graph. An arc is removed whenever $I(\{a\}, \{b\}|X \setminus \{a, b\})$ in the corresponding JPD. (This is the algorithm given on page 222 of CGH).

b. (10) U_2 starts with a graph $G = (X, \emptyset)$. U_2 iterates over each of the nodes x in the graph. For each x, U_2 searches for the smallest set $B(x)$ such that $I(\{x\}, X \setminus (\{x\} \cup B(x)) | B(x))$.[1] After U_2 identifies this set, U_2 updates the list of links L in G by setting L to $L \cup \{x - b | b \in B(x)\}$.

c. (10) U_3 factors the JPD directly via an unknown mechanism. Graph G is constructed by drawing an undirected arc between two nodes if they both appear in the same factor.

1. If more than one set B is minimal, ties are broken arbitrarily.