STA 113 Spring 2005
Exam 2

NAME

Calculator allowed, closed book.

Useful Numbers: $\Phi^{-1}(.99) = z_{.01} = 2.33, z_{.025} = 1.96, z_{.05} = 1.65, z_{.10} = 1.28$

Circle True or False.

1. Assume two random variables X, Y have joint probability mass function $p(x, y)$, with marginal mass functions p_X and p_Y and means μ_X and μ_Y respectively, and correlation coefficient $\rho_{X,Y}$.

   (a) T $E(X) = \sum_y \sum_x x\, p(x, y)$.

   (b) T $\sigma_X^2 = \sum_x \sum_y (x - \mu_X)^2 p(x, y)$.

   (c) F If $\rho_{X,Y} = 1$, then X and Y take positive values.

   (d) F If $\rho_{X,Y} = 0$, then X and Y have the same marginal distributions p_X and p_Y.

   (e) T The unit of measurement of the covariance $Cov(X, Y)$ is the product of the units of measurement of X and Y.

   (f) F If X and Y are independent, then $p(x, y) = p_X(x) + p_Y(y)$.

   (g) T If X and Y are independent, then their correlation $\rho_{X,Y} = 0$.

   (h) F If X and Y have correlation $\rho_{X,Y} = 0$, then they are independent.

Give complete solutions and show your work for problems 2-3.

2. Suppose the lifetime of two electrical parts are independent random variables T_1, T_2 with exponential distribution and mean 1 month. Find the probability $P(T_1 + T_2 < 3)$, the probability that the total of two independent lifetimes is less than 3 months.

$$\int_0^3 \int_0^{3-x} e^{-x} e^{-y} \, dy \, dx = 1 - 4e^{-3} = 0.801.$$

The sum has the Gamma(2,1) density, too.

3. Suppose the lifetimes of a sequence of electrical parts are independent random variables $T_1, T_2, T_3, \ldots$ with exponential distribution with mean $1/2$ month. Imagine using the first part until it fails, replacing it with a new part, and continuing the sequence of replacements with new parts. Compute the probability that the number of failures in the first 6 months is exactly 12.

   Let N be the number of failures in 6 months. Then $N \sim Pois(\lambda)$ and $\lambda = 6/(1/2) = 12$. Then $P(N = 12) = 12^{12}e^{-12}/12!$. On a calculator it is easiest to compute its logarithm first. The final answer is 0.114. If you use the $N(12, 12)$ density as an approximation, you would get $\frac{e^{-0}}{\sqrt{2\pi}\sqrt{12}} = 0.115$, quite a bit easier.

For problems 4-11, choose the best answer.

4. Suppose two groups of scientists each make 25 independent measurements of the pH level of a chemical compound. In the first group, each observation is subject to randomness and has the $N(5.00, \sigma = 0.1)$ distribution in units of pH. In the second group, each observation is subject to randomness and has the $N(5.00, \sigma = 0.3)$ distribution in units of pH. If $\bar{X}$ is the sample mean of the first group and $\bar{Y}$ is the sample mean of the second group, find the probability that the two sample means are within 0.05 pH of one another.

   (a) $P(|Z| < 0.7906)$ x
   (b) $P(|Z| < 0.1581)$
   (c) 0.50
   (d) 1/3
   (e) none of the above

   See Ch 5, #65.

5. Suppose the joint pmf of two discrete random variables X, Y is given by the table

$X\backslash Y$	$Y=0$	$Y=1$
$X=0$	2/3	1/6
$X=1$	1/6	0

.

Compute the correlation of X and Y.

(a) -1/5 x
(b) 0
(c) -1/36
(d) -1/6
(e) none of the above

$\rho = -1/36/(1/6 \times 5/6)$.

6. Suppose the lifetime T of a battery has a density $f(x)$ proportional to $x^2 e^{-x}$ for $x > 0$, and $f(x) = 0$ when $x \leq 0$. Then the sample mean $\bar{T}$ of a random sample of 10 such lifetimes has variance $\sigma^2_{\bar{T}}$ equal to

(a) 0.30 x
(b) 3.0
(c) 0.20
(d) 2.0
(e) none of the above

Each has variance 3×1^2, so the sample mean has variance $3/10$.

7. Assume X_1, X_2, X_3, X_4, X_5 is a random sample from some unknown distribution with mean $\mu = 10$ months, $\sigma = 1$ month, which represent lifetimes of five batteries. If $T = X_1 + X_2 + X_3 + X_4 + X_5$ is the total lifetime when they are used in sequence, find approximately the 95^{th} percentile of the distribution of T, the time below which T falls with probability .95.

   (a) 50.0 months
   (b) 54.4 months
   (c) 53.7 months x
   (d) 51.6 months
   (e) none of the above

   Use $50 + \sqrt{5 \times 1^2} \times 1.645$.

8. If a point (X, Y) is uniformly distributed on the unit circle (radius 1), find the probability that $\sqrt{X^2 + Y^2} < r$ for $0 < r < 1$.

   (a) r
   (b) r^2 x
   (c) πr^2
   (d) none of the above

9. Suppose two random variables X, Y have means $\mu_X = 0.01$ and $\mu_Y = 0.02$ (mm), standard deviations $\sigma_X = 0.01, \sigma_Y = 0.02$ (mm), and correlation $\rho_{X,Y} = .40$. Find the standard deviation of $\frac{1}{2}X + \frac{1}{2}Y$.

   (a) 0.0150
   (b) 0.0128 x
   (c) 0.0140
   (d) none of the above

10. If $X \sim \texttt{bin}(10000, .5)$, then $P(4500 \leq X \leq 5000)$ is approximately

    (a) 0.50 x
    (b) 0.01
    (c) 0.25
    (d) none of the above

    $X \sim^{approx.} N(5000, 2500)$, so an approximation is easy.

11. If Xavier arrives at a rendezvous point at a random time uniformly distributed from 12:00 to 12:15 P.M., and Yolanda arrives independently at a time uniformly distributed from 12:05 to 12:15, what is the probability that the difference of their arrival times is at most 5 minutes?

    (a) 0.50

    (b) 0.56

    (c) 0.58 x

    (d) none of the above