STA 113 Spring 2005
Exam 1

NAME

Calculator allowed, closed book.

Circle True or False.

1. Assume several pairs (x, y) include at least two different x values and two different y values so there is no division by 0 in the sample correlation.

   (a) F The sample correlation is always between 0 and 1, including the values 0 and 1.

   (b) F The fitted least-squares regression line has slope equal to the sample correlation of the x and y values.

   (c) T If the sample correlation is positive, then the slope of the fitted least-squares regression line is also positive.

   (d) F If the points (x_i, y_i) lie exactly on a line, then the sample correlation is 1.

   (e) F The units of the sample correlation are the same as the units of measurement for the x and y values.

2. Assume we have pairs of numbers (x, y) given by $(-1, -2), (0, 1), (1, 1)$ in volume units of m^3.

   (a) T The slope of the least-squares fitted line is greater than 1.2.

   (b) F The correlation of the x and y values is less than 0.8.

   (c) T The intercept of the least-squares fitted line is 0.0 m^3.

(1 point each)

Give complete solutions and show your work for problems 3-4.

3. An airline overbooks its flight with 50 seats by selling 52 tickets. Let Y be the random variable which counts the number who show up. Suppose Y has the pmf given by

y	48	49	50	51	52
$p(y)$	.4	.3	.1	.1	.1

.

(a) Find $P(Y > 50)$, the probability of an overbooking problem.

$P(Y > 50) = p(51) + p(52) = .2.$

(b) Find $E(Y)$.

$E(Y) = 48 \times .4 + \ldots + 52 \times .1 = 49.2$ seats.

(c) If each arriving passenger without a seat costs \$400, find $E(C)$, the expected cost of the overbooking.
$E(C) = \$400 \times .1 + \$800 \times .1 = \$120.$

(5 points)

4. Assume a packet arrives on an IP network in a time interval of 10^{-4}s with probability $p > 0$, and such arrivals occur independently from one interval to the next and one-at-a-time like coin flips. Also, assume the expected number of intervals until the next arrival is 100.

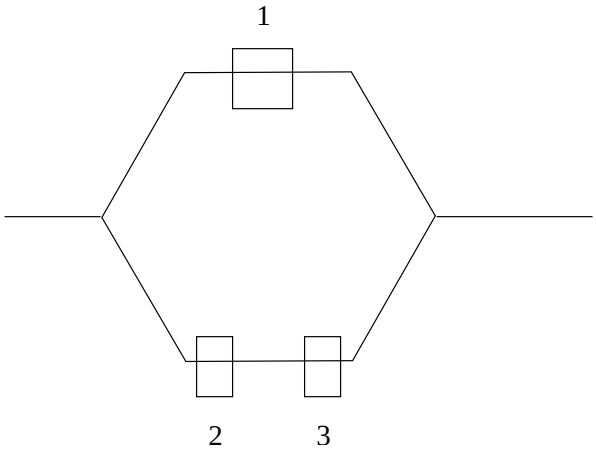
   Compute the probability that the number N of packets in the next 100 intervals will be 2 or more.

   $N \sim \texttt{bin}(100, p)$ with $p = 1/100$. So $P(N \geq 2) = 1 - P(N \leq 1) \approx 1 - (e^{-1} + e^{-1}) = .264$.

(5 points)

Choose the best answer.

5. In the system below, each component functions independently of the others with probability $p = .9$. What is the probability that the entire system does not work?



   (a) 0.981
   (b) 0.019 x
   (c) 0.290
   (d) none of the above

   See #78, Ch 2. $(.1 \times (.1 + .1 - .1^2))$

6. A personnel director interviewing 11 engineers for four job openings has scheduled 6 interviews for the first day. The interview randomly selects six candidates for the first day. What is the probability that all five of the top five candidates get interviewed the first day?

   (a) 0.013 x
   (b) 0.833
   (c) 0.168
   (d) none of the above

   See #68, Ch 3. (Hypergeometric: $\binom{5}{5}\binom{6}{1}/\binom{11}{6}$)

7. A very large batch of components has arrived at a distributor. The distributor decides to randomly select 10 components and to accept the batch only if the number of defective components in the sample is at most 1.

   What is the probability that the batch will be accepted if the proportion of defectives is 0.10?

   (a) 0.35
   (b) 0.74 x
   (c) 0.10
   (d) none of the above

   See #54, Ch 3. (Number of defectives $N \sim \texttt{bin}(10, .1)$.)

8. In Texas Hold'em I have two hearts and the flop has two hearts as well. What is the probability that I do not get the flush (five hearts) that I am hoping for from the remaining two cards?

   (a) 0.67
   (b) 0.66
   (c) 0.65 x
   (d) none of the above

   $\binom{52-13-1}{2} / \binom{52-5}{2}$

9. The number of people arriving for treatment at an emergency room can be modeled by a Poisson process with a rate parameter of 4/hour. What is the probability that at least two people arrive in the next 1/2 hour?

   (a) 0.91

   (b) 0.59 x

   (c) 0.32

   (d) none of the above

   See #82, Ch 3.

10. A box contains 4 blue light bulbs, 5 green light bulbs, and 6 red bulbs. If bulbs are selected in random order, what is the probability that at least two will be selected before a blue one is drawn?

    (a) 0.52 x

    (b) 0.54

    (c) 0.73

    (d) none of the above

    See #18, Ch 2. $(11/15 \times 10/14)$

11. A disease occurs in 1% of a population. A test for the disease is 99% accurate, meaning that the conditional probability that a randomly selected person tests positive given that he/she has the disease is 0.99, and the conditional probability that a randomly selected person tests negative given that he/she does not have the disease is 0.99. Find the conditional probability that a randomly selected person has the disease, given that he/she tests positive.

    (a) 0.99

    (b) 0.50 x

    (c) 0.85

    (d) none of the above

    See #64, Ch 2. (Use Bayes' Rule to get $P(D \mid +)$.)

12. A test for the presence of a certain disease has a probability of .20 of giving a false-positive reading, and probability .10 of giving a false-negative result. Suppose that four individuals are tested, of which exactly two have the disease. Let X be the number of positive readings in the results. What is $P(X = 1)$?

    (a) 0.58

    (b) 0.50

    (c) 0.53

    (d) none of the above x

    See #109, Ch 3.

(4 points each)