Practice Final Examination

Mth 136 = Sta 114

Wednesday, 2000 April 26, 2:20 – 3:00 pm

This is a closed-book examination so please do not refer to your notes, the text, or to any other books. You may use a two-sided single sheet of *your own* notes, if you wish, but you may not share materials. A normal distribution table, the PDF handout, and a blank worksheet are attached to the exam. If you don't understand something in one of the questions feel free to ask me, but please do not talk to each other. You may use a calculator but not a laptop computer.

You must **show** your **work** to get partial credit. Unsupported answers are not acceptable, even if they are correct. Please give all numerical answers as fractions **in lowest terms** (simplify!) or as decimals correct to **four places**. You should spend about 10–15 minutes on each problem. It is to your advantage to write your solutions as clearly as possible, since I cannot give credit for solutions I do not understand. Good luck.

Cheating on exams is a breach of trust with classmates and faculty, and will not be tolerated. After completing the exam please acknowledge the Duke Honor Code:

I have neither given nor received any unauthorized aid on this exam.

Signature: ________________________

1.	/25
2.	/25
3.	/25
⋮	⋮
Total:	/200

Problem 1: Suppose that the probability θ that a car will require service in its first 10 000 miles is unknown, but that an automobile manufacturer has data about 200 similar cars sold in January, 1994: of these 200, 22 required service during the first 10 000 miles.

a) (10) If we use a uniform reference prior density $\pi(\theta) = 1$, what is the posterior distribution for θ?

b) (5) Under the same reference prior distribution, what is the posterior probability of the hypothesis $H_0 : \theta \le 0.10$? Give your answer in the form of an integral OR an `S-Plus` command.

c) (10) Find the significance level (P-value) for a test of that same hypothesis, in the form of an integral OR an `S-Plus` command. It turns out that the numerical answer is $P = 0.3516478$; would you accept or reject H_0 at level $\alpha = 0.05$?

XC) (+5) Give the approximate numericals answer to b) and c) above, by using normal approximations.

Problem 2: `S-Plus` yields the following output for a linear model of log price *vs.* mileage for used Mercedes cars. Recall that prices are in (British) Pounds Sterling, while mileage is in thousands of miles.

```
> mb.fit1 <- lm(log(Price) ~ Mile, data=merc)
> summary(mb.fit1)

Call: lm(formula = log(Price) ~ Mile, data = merc)
Residuals:
     Min      1Q Median     3Q     Max
 -0.5403 -0.2627 0.0327 0.2280  0.6375

Coefficients:
                 Value Std. Error    t value
(Intercept)    9.89740    0.08568  115.50885
       Mile   -0.01680    0.00366   -4.59026
                  Pr(>|t|)
(Intercept)    0.000000000
       Mile    0.000028356

Residual standard error: 0.3158 on 52 degrees of freedom
Multiple R-Squared: 0.2883
F-statistic: 21.07 on 1 and 52 deg fdm, p-value 0.000028356
```

a) (5) If a particular used Mercedes could be sold for about £10 000, how much would it be worth if it were driven another 1 000 miles?

b) (10) Give a 95% confidence interval for the coefficient β_1 of mileage in the regression equation. Recall that the t distribution is essentially identical to the normal distribution, whenever the number of degrees of freedom exceed thirty or so.

c) (10) The regression `mb.fit2 <- lm(log(Price) ~ Mile+Age+Mod)` yields different estimates

```
> summary(mb.fit2)

Call: lm(formula = log(Price) ~  Mile + Age + Mod, data = merc)
Residuals:
     Min       1Q   Median      3Q    Max
 -0.1963 -0.06524 0.001757 0.06455  0.244

Coefficients:
                  Value    Std. Err   t value  Pr(>|t|)
(Intercept)   10.00669    0.03768  265.54618  0.0000
       Mile   -0.00310    0.00172   -1.80500  0.0774
        Age   -0.07574    0.01161   -6.52006  0.0000
       Mod1   -0.04107    0.03371   -1.21841  0.2291
       Mod2   -0.02470    0.01537   -1.60691  0.1147
       Mod3   -0.07689    0.00866   -8.87672  0.0000
       Mod4   -0.13000    0.00660  -19.67022  0.0000

Residual standard error: 0.0995932 on 47 degrees of freedom
Multiple R-Squared: 0.936057
F-statistic: 114.6 on 6 and 47 deg fdm, p-value is 0
```

This time the estimated mileage coefficient is only $\hat{\beta}_1 = -.0031$ (five times less than the -0.0168 from the straight-line model above) and the P-value for a test of $\beta_1 = 0$ is $P = .0774$, suggesting that perhaps mileage isn't so important in predicting price. **How is this possible**? Why does mileage seem so important in predicting price in Fit 1 but not in Fit 2?

Problem 3: An astronomer is testing the hypothesis that an interstellar cloud exists in a certain patch of sky. If true, starlight should be attenuated there and the patch should appear darker than typical. The number of detectable stars in any patch of sky of solid angle[1] A has a Poisson distribution with mean $\lambda = \theta A$, with $\theta = 10^7$ for "typical" patches of sky, so her hypothesis becomes $H_0 : \theta < 10^7$. With a solid angle of $A = 10^{-5}$, she observes $y = 84$ stars.

a) (5) Find the likelihood function $f(y \mid A, \theta)$.

b) (5) Find the maximum likelihood estimate $\hat{\theta}$ (for this A and y).

c) (5) With a reference prior distribution $\pi(\theta) = 1/\theta$, find the posterior distribution for θ.

d) (5) Give the posterior mean and variance of this distribution (note this does *not* require any integration).

e) (5) In fact the distribution for θ is well approximated by a normal distribution with the same mean and variance. Using this approximation, give the approximate probability that the patch is dark, i.e., that $\theta < 10^7$.

[1] You don't need to know what "solid angle" means to do this problem— but if you're curious, it's the area of the projection of the "patch" onto the unit sphere... so a one-degree square has solid angle of approximately $4\pi \times (1/360)^2 \approx 10^{-4}$.

Extra worksheet, if needed:

Name: ________________

$$\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-t^2/2}\, dt:$$

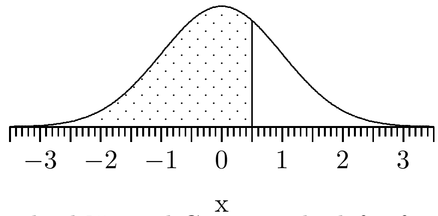

Table 5.1 Area $\Phi(x)$ under the Standard Normal Curve to the left of x.

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998