

STA205 Homework 2

Due: Sep 15 2009

1. Let $\Omega = (0, 1] \times (0, 1]$, and consider the following collections of subsets of Ω :

$$\mathcal{A} = \{(0, a] \times (0, b] : 0 < a, b \leq 1\}$$

- (a) Is \mathcal{A} a π -system? Why or why not?
- (b) Is \mathcal{A} a λ -system? Why or why not?

2. Consider the following collection of subsets of the real line:

$$\mathcal{B} = \{(-\infty, b], b \in \mathbb{R}\}$$

- (a) Show that \mathcal{B} is a π -system, but not a λ -system.
- (b) What is the λ -system generated by \mathcal{B} ?

3. Let \mathcal{C} be a nonempty collection of subsets of Ω . Find two probability measures $\mathbf{P}_1, \mathbf{P}_2$ on Ω that agree on \mathcal{C} , but not on $\mathcal{A}(\mathcal{C})$, which is the minimal field over \mathcal{C} .

4. Let $\mathcal{C} = \{C_j\}$ be the collection of subsets $C_j \equiv (0, j/3]$ of $\Omega = (0, 1]$, for $j = 0, 1, 2, 3$.

- (a) Describe in detail the smallest π -system $\pi(\mathcal{C})$ containing \mathcal{C} . How many elements does $\pi(\mathcal{C})$ include?
- (b) Describe in detail the smallest λ -system $\lambda(\mathcal{C})$ containing \mathcal{C} . How many elements does $\lambda(\mathcal{C})$ include?
- (c) Suppose $\mathbf{P}_1, \mathbf{P}_2$, two probability measures on Ω , agree on $\pi(\mathcal{C})$, will they agree on $\lambda(\mathcal{C})$? Why or why not?
- (d) Find one or two examples to show $\mathbf{P}_1, \mathbf{P}_2$ agree or not on $\lambda(\mathcal{C})$.