# Midterm Examination # 2

Mth 136 = Sta 114

Monday, April 16, 2009
2:50 – 4:05 pm
Version *b*

- This is a **closed book** exam— put your books on the floor.
- You may use a calculator and **one page** of your own notes.
- Do not share calculators or notes.
- **Show** your **work**. Neatness counts. Boxing answers helps.
- Numerical answers: **four significant digits** or simplified exact solution (e.g., fractions **in lowest terms**). **Simplify** *all* answers for full credit.
- Extra worksheet and pdf & distribution tables are attached.

Cheating on exams is a breach of trust with classmates and faculty that will not be tolerated. After completing the exam please acknowledge that you have adhered to the Duke Community Standard:

- I will not lie, cheat, or steal in my academic endeavors;
- I will conduct myself honorably in all my endeavors; and
- I will act if the Standard is compromised.

Signature: ______________________

Print **Name**: ______________________

| | |
|---|---|
| 1. | /20 |
| 2. | /20 |
| 3. | /20 |
| 4. | /20 |
| 5. | /20 |
| Total: | /100 |

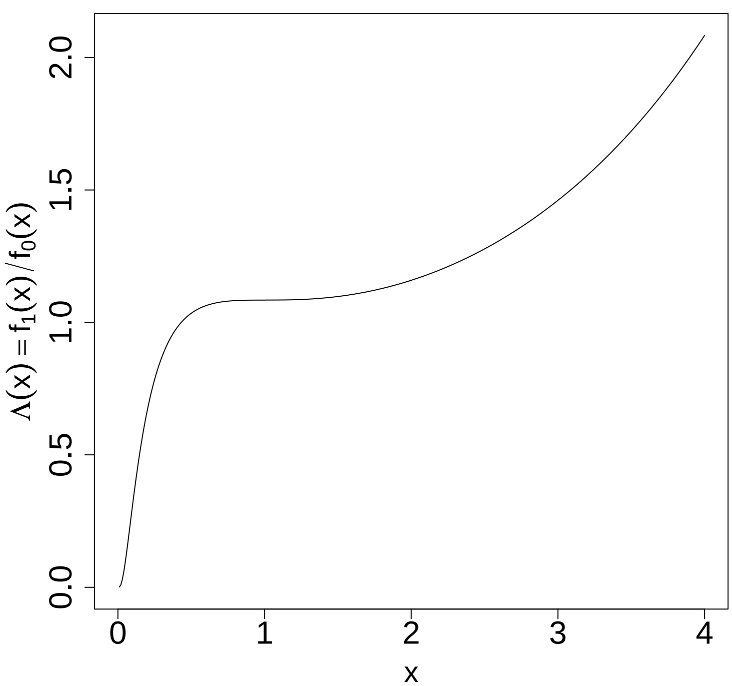
**Problem 1**: Skyler thinks $X$ has a standard exponential distribution with density funcition

$$f_0(x) = e^{-x}\mathbf{1}_{\{x>0\}},$$

while Terry thinks it has a lognormal distribution (so $Y \equiv \log X$ has a standard Normal distribution $Y \sim \mathsf{No}(0,1)$, $X = e^Y$) with density function

$$f_1(x) = \frac{1}{x\sqrt{2\pi}} e^{-(\log x)^2/2}\mathbf{1}_{\{x>0\}}.$$

When they disagree about things, Skyler is right about 2/3 of the time. Here's a plot of the likelihood ratio against $H_0$, $\Lambda(x) = f_1(x)/f_0(x)$, vs. $x$:



**Problem 1** (cont):

a) (6) Prove that the likelihood ratio increases monotonically on all of $0 < x < \infty$.

b) (6) Based on the information above and the single observation $X = 3$, what is the (posterior) probability that Skyler was right? Give answer to four significant digits (or exact answer), and **simplify**:

$\mathsf{P}[$ Skyler correct $\mid X = 3] =$

**Problem 1** (cont):

c) (6) Find the $P$-value for likelihood ratio test of $H_0 : X \sim f_0(x)$ against alternative $H_1 : X \sim f_1(x)$ for the single observation $X = 3$ (as usual, exact or 4 sig digs:

$P =$

c) (1) Would you Accept or Reject $H_0$ at level $\alpha = 0.05$?

d) (1) Is the test in c) above UMP? Circle one: Y N
Why or why not?

**Problem 2**: The $n = 9$ independent random variables $\{X_i\}_{i \le n}$ are a simple random sample from the Normal distribution $X_i \sim \mathsf{No}(\mu, \sigma^2)$, with variance $\sigma^2$ and mean $\mu$.

a) (7) With $\sigma^2$ unknown, describe the Rejection Region $\mathcal{R}$ for a two-sided test of size $\alpha = 0.05$ of the hypothesis

$$H_0: \ \mu = 17.5 \qquad vs. \qquad H_1: \ \mu \neq 17.5$$

Reject $H_0$ if:

b) (7) Same question, if variance of $\{X_i\}$ is known to be $\sigma^2 = 16$:

Reject $H_0$ if:

c) (6) What is the *power* $\pi(\mu)$ of the test in part b) above, for $\mu = 20$?

$\pi(20) =$ ______________

**Problem 3**: Once again the $n = 9$ independent random variables $\{X_i\}_{i \le n}$ are a simple random sample from the Normal distribution $X_i \sim \mathsf{No}(\mu, \sigma^2)$, with variance $\sigma^2$ and mean $\mu$, but now we have some data. The values of a few statistics from this random sample are:

$$S(\mathbf{x}) = \sum_{i \le n} X_i = 180 \qquad T(\mathbf{x}) = \min_{i \le n} X_i = 1$$
$$V(\mathbf{x}) = \sum_{i \le n} (X_i - \bar{X})^2 = 72 \qquad W(\mathbf{x}) = \text{Median}(\{X_i\}) = 24$$

a) (7) With $\sigma^2$ unknown, give the $P$-value for a two-sided test of the hypothesis

$$H_0: \ \mu = 17.5 \qquad vs. \qquad H_1: \ \mu \neq 17.5$$

$P =$

b) (7) Same question, if variance of $\{X_i\}$ is known to be $\sigma^2 = 16$:

$P =$

c) (6) Are these answers consistent with your answers to parts a) and b) of Problem 2? Y N Explain.

**Problem 4**: For some parameter value $0 < \theta < \infty$, the three random variables $\{X_i\}_{1 \le i \le 3}$ are uniformly distributed on the interval $[0, \theta]$. A few statistics from this random sample to consider are:

$$S(\mathbf{x}) = \sum X_i \quad T(\mathbf{x}) = \sum (X_i - \bar{X})^2 \quad V(\mathbf{x}) = \min X_i \quad W(\mathbf{x}) = \max X_i$$

a) (6) If $\theta$ is known to be one of $\{\theta_0 = 2,\ \theta_1 = 3\}$, with prior probabilities $\xi_0 = 0.8$ and $\xi_1 = 0.2$ respectively, find the posterior probability indicated, as a function of one or more of $S, T, V, W$. Simplify!

$\xi(\theta = \theta_0 \mid \mathbf{x}) =$

b) (6) If (instead) $\theta$ might be any number in $[1, \infty)$, with a Pareto prior distribution $\theta \sim \mathsf{Pa}(\alpha, \epsilon)$ with density function

$$\xi(\theta) = \alpha\epsilon^\alpha / \theta^{\alpha+1} \mathbf{1}_{\{\theta \ge \epsilon\}}$$

with $\alpha = 1$ and $\epsilon = 1$, find the posterior density function for $\theta$, correctly for all $\theta \in \mathbb{R}$. If possible, give your answer using one or more of $S, T, V, W$. Simplify!

$\xi(\theta \mid \mathbf{x}) =$

**Problem 4** (cont):

c) (8) Now suppose we observe data $\mathbf{x} = \{0.60, 1.75, 1.25\}$ and we wish to make a decision about the hypothesis $H_0: \ \theta \le 2$ with alternative $H_1: \ \theta > 2$, using the Pareto prior from b) above. If we lose \$1 for choosing $H_0$ when in fact $H_1$ is true, and we lose \$$w$ for choosing $H_1$ when in fact $H_0$ is true, what decision $\delta(\mathbf{x})$ $(= 0 \text{ or } 1)$ should we make to minimize our expected loss? This may depend on $w$, of course... Show your work.

$\delta(\mathbf{x}) =$

**Problem 5**: Part (a) of this problem is completely unrelated to parts (b) and (c).

a) (10) In a famous experiment, Gregor Mendel observed the shape and color of peas that resulted from certain crossbreedings. The reported results of one such experiment were:

| | | | |
|---|---|---|---|
| round yellow | 315 | wrinkled yellow | 101 |
| round green | 108 | wrinkled green | 32 |

According to his theory, the frequencies of these four classifications should be in the ratio 9 : 3 : 3 : 1. What do you conclude from a $\chi^2$ test? Please give the value "$\chi^2$" of the test statistic (called "$Q$" by DeGroot) used for $\chi^2$ tests, its number of degrees of freedom "d.f.", and the $P$-value for the hypothesis that Mendel's theory is correct, "$P$."

$\chi^2 =$ ____________ d.f. = ____________ $P =$ ____________

**Problem 5** (cont):

b) (5) The 10 random variables $\{X_i\}$ are independent and take nonnegative integer values. Describe how you would use a generalized likelihood ratio test (GLRT) to test the hypothesis $H_0$ that these come from a Poisson distribution with mean $\lambda = 3$ against the composite alternative $H_1$ that they come from a Poisson distribution with some other (unspecified) mean. What statistics would you need from the data? Give a test statistic $T(\mathbf{x})$ such that your recommended test would reject $H_0$ for large values of $T$.

$$H_0: \ \{X_i\}_{i \le n} \sim \mathsf{Po}(\lambda) \quad \text{for } \lambda = 3 \quad vs.$$
$$H_1: \ \{X_i\}_{i \le n} \sim \mathsf{Po}(\lambda) \quad \text{for } \lambda \neq 3.$$

(xc) Find the approximate probability distribution of $T$, if $H_0$ is true:

**Problem 5** (cont):

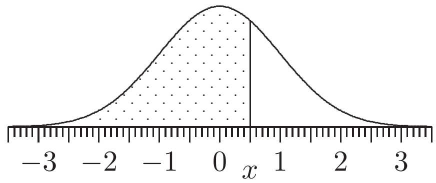
c) (5) The 10 random variables $\{X_i\}$ are (still) independent and take nonnegative integer values. Describe how you would use a GLRT to test the hypothesis $H_0$ that $\{X_i\}$ come from a Poisson distribution (with unspecified mean $\lambda$) against the alternative $H_1$ that they come from a Binomial distribution with $n = 5$ (but $p$ unspecified):

$$\begin{aligned} H_0 :&\ \{X_i\}_{i \le n} \sim \mathsf{Po}(\lambda) \quad \text{for some } \lambda \in [0, \infty) \quad \textit{vs.} \\ H_1 :&\ \{X_i\}_{i \le n} \sim \mathsf{Bi}(5, p) \quad \text{for some } p \in [0, 1] \end{aligned}$$

What statistics would you need from the data? Give a test statistic $T(\mathbf{x})$ such that your recommended test would reject $H_0$ for large values of $T$.

Extra worksheet, if needed:

$$\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-z^2/2}\, dz:$$




Area $\Phi(x)$ under the Standard Normal Curve to the left of $x$.

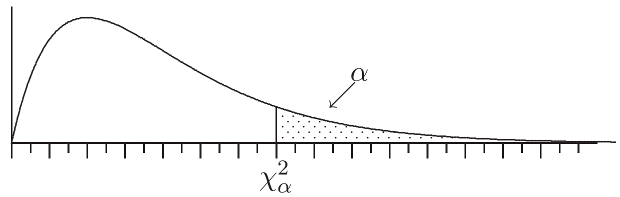
| $x$ | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| .0 | .5000 | .5040 | .5080 | .5120 | .5160 | .5199 | .5239 | .5279 | .5319 | .5359 |
| .1 | .5398 | .5438 | .5478 | .5517 | .5557 | .5596 | .5636 | .5675 | .5714 | .5753 |
| .2 | .5793 | .5832 | .5871 | .5910 | .5948 | .5987 | .6026 | .6064 | .6103 | .6141 |
| .3 | .6179 | .6217 | .6255 | .6293 | .6331 | .6368 | .6406 | .6443 | .6480 | .6517 |
| .4 | .6554 | .6591 | .6628 | .6664 | .6700 | .6736 | .6772 | .6808 | .6844 | .6879 |
| .5 | .6915 | .6950 | .6985 | .7019 | .7054 | .7088 | .7123 | .7157 | .7190 | .7224 |
| .6 | .7257 | .7291 | .7324 | .7357 | .7389 | .7422 | .7454 | .7486 | .7517 | .7549 |
| .7 | .7580 | .7611 | .7642 | .7673 | .7704 | .7734 | .7764 | .7794 | .7823 | .7852 |
| .8 | .7881 | .7910 | .7939 | .7967 | .7995 | .8023 | .8051 | .8078 | .8106 | .8133 |
| .9 | .8159 | .8186 | .8212 | .8238 | .8264 | .8289 | .8315 | .8340 | .8365 | .8389 |
| 1.0 | .8413 | .8438 | .8461 | .8485 | .8508 | .8531 | .8554 | .8577 | .8599 | .8621 |
| 1.1 | .8643 | .8665 | .8686 | .8708 | .8729 | .8749 | .8770 | .8790 | .8810 | .8830 |
| 1.2 | .8849 | .8869 | .8888 | .8907 | .8925 | .8944 | .8962 | .8980 | .8997 | .9015 |
| 1.3 | .9032 | .9049 | .9066 | .9082 | .9099 | .9115 | .9131 | .9147 | .9162 | .9177 |
| 1.4 | .9192 | .9207 | .9222 | .9236 | .9251 | .9265 | .9279 | .9292 | .9306 | .9319 |
| 1.5 | .9332 | .9345 | .9357 | .9370 | .9382 | .9394 | .9406 | .9418 | .9429 | .9441 |
| 1.6 | .9452 | .9463 | .9474 | .9484 | .9495 | .9505 | .9515 | .9525 | .9535 | .9545 |
| 1.7 | .9554 | .9564 | .9573 | .9582 | .9591 | .9599 | .9608 | .9616 | .9625 | .9633 |
| 1.8 | .9641 | .9649 | .9656 | .9664 | .9671 | .9678 | .9686 | .9693 | .9699 | .9706 |
| 1.9 | .9713 | .9719 | .9726 | .9732 | .9738 | .9744 | .9750 | .9756 | .9761 | .9767 |
| 2.0 | .9772 | .9778 | .9783 | .9788 | .9793 | .9798 | .9803 | .9808 | .9812 | .9817 |
| 2.1 | .9821 | .9826 | .9830 | .9834 | .9838 | .9842 | .9846 | .9850 | .9854 | .9857 |
| 2.2 | .9861 | .9864 | .9868 | .9871 | .9875 | .9878 | .9881 | .9884 | .9887 | .9890 |
| 2.3 | .9893 | .9896 | .9898 | .9901 | .9904 | .9906 | .9909 | .9911 | .9913 | .9916 |
| 2.4 | .9918 | .9920 | .9922 | .9925 | .9927 | .9929 | .9931 | .9932 | .9934 | .9936 |
| 2.5 | .9938 | .9940 | .9941 | .9943 | .9945 | .9946 | .9948 | .9949 | .9951 | .9952 |
| 2.6 | .9953 | .9955 | .9956 | .9957 | .9959 | .9960 | .9961 | .9962 | .9963 | .9964 |
| 2.7 | .9965 | .9966 | .9967 | .9968 | .9969 | .9970 | .9971 | .9972 | .9973 | .9974 |
| 2.8 | .9974 | .9975 | .9976 | .9977 | .9977 | .9978 | .9979 | .9979 | .9980 | .9981 |
| 2.9 | .9981 | .9982 | .9982 | .9983 | .9984 | .9984 | .9985 | .9985 | .9986 | .9986 |
| 3.0 | .9987 | .9987 | .9987 | .9988 | .9988 | .9989 | .9989 | .9989 | .9990 | .9990 |
| 3.1 | .9990 | .9991 | .9991 | .9991 | .9992 | .9992 | .9992 | .9992 | .9993 | .9993 |
| 3.2 | .9993 | .9993 | .9994 | .9994 | .9994 | .9994 | .9994 | .9995 | .9995 | .9995 |
| 3.3 | .9995 | .9995 | .9995 | .9996 | .9996 | .9996 | .9996 | .9996 | .9996 | .9997 |
| 3.4 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9998 |

$\Phi(0.6745) = 0.75$ $\Phi(1.6449) = 0.95$ $\Phi(2.3263) = 0.99$ $\Phi(3.0902) = 0.999$

$\Phi(1.2816) = 0.90$ $\Phi(1.9600) = 0.975$ $\Phi(2.5758) = 0.995$ $\Phi(3.2905) = 0.9995$

## Critical Values for $\chi^2$

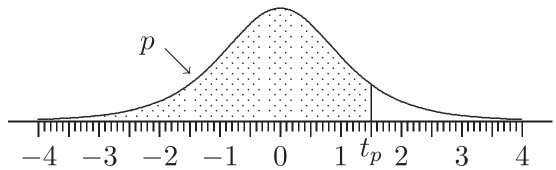
$$\alpha = \int_{\chi^2_\alpha}^{\infty} c\, x^{\nu/2-1} e^{-x/2}\, dx$$




| $\nu$ | $\chi^2_{.50}$ | $\chi^2_{.25}$ | $\chi^2_{.10}$ | $\chi^2_{.05}$ | $\chi^2_{.025}$ | $\chi^2_{.01}$ | $\chi^2_{.005}$ | $\chi^2_{.001}$ | $\chi^2_{.0005}$ | $\chi^2_{.0001}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.4549 | 1.3233 | 2.7055 | 3.8415 | 5.0239 | 6.6349 | 7.87940 | 10.8276 | 12.1157 | 15.1367 |
| 2 | 1.3863 | 2.7726 | 4.6052 | 5.9915 | 7.3778 | 9.2103 | 10.5966 | 13.8155 | 15.2018 | 18.4207 |
| 3 | 2.3660 | 4.1083 | 6.2514 | 7.8147 | 9.3484 | 11.3449 | 12.8382 | 16.2662 | 17.7300 | 21.1075 |
| 4 | 3.3567 | 5.3853 | 7.7794 | 9.4877 | 11.1433 | 13.2767 | 14.8603 | 18.4668 | 19.9974 | 23.5127 |
| 5 | 4.3515 | 6.6257 | 9.2364 | 11.0705 | 12.8325 | 15.0863 | 16.7496 | 20.5150 | 22.1053 | 25.7448 |
| 6 | 5.3481 | 7.8408 | 10.6446 | 12.5916 | 14.4494 | 16.8119 | 18.5476 | 22.4577 | 24.1028 | 27.8563 |
| 7 | 6.3458 | 9.0371 | 12.0170 | 14.0671 | 16.0128 | 18.4753 | 20.2777 | 24.3219 | 26.0178 | 29.8775 |
| 8 | 7.3441 | 10.219 | 13.3616 | 15.5073 | 17.5345 | 20.0902 | 21.9550 | 26.1245 | 27.8680 | 31.8276 |
| 9 | 8.3428 | 11.389 | 14.6837 | 16.9190 | 19.0228 | 21.6660 | 23.5894 | 27.8772 | 29.6658 | 33.7199 |
| 10 | 9.3418 | 12.549 | 15.9872 | 18.3070 | 20.4831 | 23.2092 | 25.1882 | 29.5883 | 31.4198 | 35.5640 |
| 11 | 10.341 | 13.701 | 17.2750 | 19.6751 | 21.9200 | 24.7249 | 26.7568 | 31.2641 | 33.1366 | 37.3670 |
| 12 | 11.340 | 14.845 | 18.5493 | 21.0260 | 23.3366 | 26.2169 | 28.2995 | 32.9095 | 34.8213 | 39.1344 |
| 13 | 12.340 | 15.984 | 19.8119 | 22.3620 | 24.7356 | 27.6882 | 29.8195 | 34.5282 | 36.4778 | 40.8707 |
| 14 | 13.339 | 17.117 | 21.0641 | 23.6848 | 26.1189 | 29.1412 | 31.3193 | 36.1233 | 38.1094 | 42.5793 |
| 15 | 14.339 | 18.245 | 22.3071 | 24.9958 | 27.4884 | 30.5779 | 32.8013 | 37.6973 | 39.7188 | 44.2632 |
| 16 | 15.338 | 19.369 | 23.5418 | 26.2962 | 28.8453 | 31.9999 | 34.2672 | 39.2524 | 41.3081 | 45.9249 |
| 17 | 16.338 | 20.489 | 24.7690 | 27.5871 | 30.1910 | 33.4087 | 35.7185 | 40.7902 | 42.8792 | 47.5664 |
| 18 | 17.338 | 21.605 | 25.9894 | 28.8693 | 31.5264 | 34.8053 | 37.1565 | 42.3124 | 44.4338 | 49.1894 |
| 19 | 18.338 | 22.718 | 27.2036 | 30.1435 | 32.8523 | 36.1909 | 38.5823 | 43.8202 | 45.9731 | 50.7955 |
| 20 | 19.337 | 23.828 | 28.4120 | 31.4104 | 34.1696 | 37.5662 | 39.9968 | 45.3147 | 47.4985 | 52.3860 |
| 21 | 20.337 | 24.935 | 29.6151 | 32.6706 | 35.4789 | 38.9322 | 41.4011 | 46.7970 | 49.0108 | 53.9620 |
| 22 | 21.337 | 26.039 | 30.8133 | 33.9244 | 36.7807 | 40.2894 | 42.7957 | 48.2679 | 50.5111 | 55.5246 |
| 23 | 22.337 | 27.141 | 32.0069 | 35.1725 | 38.0756 | 41.6384 | 44.1813 | 49.7282 | 52.0002 | 57.0746 |
| 24 | 23.337 | 28.241 | 33.1962 | 36.4150 | 39.3641 | 42.9798 | 45.5585 | 51.1786 | 53.4788 | 58.6130 |
| 25 | 24.337 | 29.339 | 34.3816 | 37.6525 | 40.6465 | 44.3141 | 46.9279 | 52.6197 | 54.9475 | 60.1403 |
| 26 | 25.336 | 30.435 | 35.5632 | 38.8851 | 41.9232 | 45.6417 | 48.2899 | 54.0520 | 56.4069 | 61.6573 |
| 27 | 26.336 | 31.528 | 36.7412 | 40.1133 | 43.1945 | 46.9629 | 49.6449 | 55.4760 | 57.8576 | 63.1645 |
| 28 | 27.336 | 32.620 | 37.9159 | 41.3371 | 44.4608 | 48.2782 | 50.9934 | 56.8923 | 59.3000 | 64.6624 |
| 29 | 28.336 | 33.711 | 39.0875 | 42.5570 | 45.7223 | 49.5879 | 52.3356 | 58.3012 | 60.7346 | 66.1517 |
| 30 | 29.336 | 34.800 | 40.2560 | 43.7730 | 46.9792 | 50.8922 | 53.6720 | 59.7031 | 62.1619 | 67.6326 |
| 40 | 39.336 | 45.616 | 51.8051 | 55.7585 | 59.3417 | 63.6907 | 66.7660 | 73.4020 | 76.0946 | 82.0623 |
| 50 | 49.335 | 56.334 | 63.1671 | 67.5048 | 71.4202 | 76.1539 | 79.4900 | 86.6608 | 89.5605 | 95.9687 |
| 60 | 59.335 | 66.981 | 74.3970 | 79.0819 | 83.2977 | 88.3794 | 91.9517 | 99.6072 | 102.695 | 109.503 |
| 70 | 69.335 | 77.577 | 85.5270 | 90.5312 | 95.0232 | 100.425 | 104.215 | 112.317 | 115.578 | 122.755 |
| 80 | 79.334 | 88.130 | 96.5782 | 101.879 | 106.629 | 112.329 | 116.321 | 124.839 | 128.261 | 135.782 |
| 90 | 89.334 | 98.650 | 107.565 | 113.145 | 118.136 | 124.116 | 128.299 | 137.208 | 140.782 | 148.627 |
| 100 | 99.334 | 109.14 | 118.498 | 124.342 | 129.561 | 135.807 | 140.169 | 149.449 | 153.167 | 161.319 |

## Critical Values for Student's $t$

$$p = \int_{-\infty}^{t_p} c \, \frac{dt}{(1 + t^2/\nu)^{(\nu+1)/2}}$$




| $\nu$ | $t_{.60}$ | $t_{.70}$ | $t_{.80}$ | $t_{.85}$ | $t_{.90}$ | $t_{.95}$ | $t_{.975}$ | $t_{.99}$ | $t_{.995}$ | $t_{.999}$ | $t_{.9995}$ | $t_{.9999}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.325 | 0.727 | 1.376 | 1.9626 | 3.078 | 6.314 | 12.76 | 31.82 | 63.66 | 318.3 | 636.6 | 3183. |
| 2 | 0.289 | 0.617 | 1.061 | 1.3862 | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 | 22.33 | 31.60 | 70.70 |
| 3 | 0.277 | 0.584 | 0.978 | 1.2498 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 | 10.22 | 12.92 | 22.20 |
| 4 | 0.271 | 0.569 | 0.941 | 1.1896 | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 | 7.173 | 8.610 | 13.03 |
| 5 | 0.267 | 0.559 | 0.920 | 1.1558 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 | 5.893 | 6.869 | 9.678 |
| 6 | 0.265 | 0.553 | 0.906 | 1.1342 | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 | 5.208 | 5.959 | 8.025 |
| 7 | 0.263 | 0.549 | 0.896 | 1.1192 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 | 4.785 | 5.408 | 7.063 |
| 8 | 0.262 | 0.546 | 0.889 | 1.1081 | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 | 4.501 | 5.041 | 6.442 |
| 9 | 0.261 | 0.543 | 0.883 | 1.0997 | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 | 4.297 | 4.781 | 6.010 |
| 10 | 0.260 | 0.542 | 0.879 | 1.0931 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 | 4.144 | 4.587 | 5.694 |
| 11 | 0.260 | 0.540 | 0.876 | 1.0877 | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 | 4.025 | 4.437 | 5.453 |
| 12 | 0.259 | 0.539 | 0.873 | 1.0832 | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 | 3.930 | 4.318 | 5.263 |
| 13 | 0.259 | 0.538 | 0.870 | 1.0795 | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 | 3.852 | 4.221 | 5.111 |
| 14 | 0.258 | 0.537 | 0.868 | 1.0763 | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 | 3.787 | 4.140 | 4.985 |
| 15 | 0.258 | 0.536 | 0.866 | 1.0735 | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 | 3.733 | 4.073 | 4.880 |
| 16 | 0.258 | 0.535 | 0.865 | 1.0711 | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 | 3.686 | 4.015 | 4.791 |
| 17 | 0.257 | 0.534 | 0.863 | 1.0690 | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 | 3.646 | 3.965 | 4.714 |
| 18 | 0.257 | 0.534 | 0.862 | 1.0672 | 1.330 | 1.734 | 2.101 | 2.552 | 2.878 | 3.610 | 3.922 | 4.648 |
| 19 | 0.257 | 0.533 | 0.861 | 1.0655 | 1.328 | 1.729 | 2.093 | 2.539 | 2.861 | 3.579 | 3.883 | 4.590 |
| 20 | 0.257 | 0.533 | 0.860 | 1.0640 | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 | 3.552 | 3.85 | 4.539 |
| 21 | 0.257 | 0.532 | 0.859 | 1.0627 | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 | 3.527 | 3.819 | 4.493 |
| 22 | 0.256 | 0.532 | 0.858 | 1.0614 | 1.321 | 1.717 | 2.074 | 2.508 | 2.819 | 3.505 | 3.792 | 4.452 |
| 23 | 0.256 | 0.532 | 0.858 | 1.0603 | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 | 3.485 | 3.768 | 4.415 |
| 24 | 0.256 | 0.531 | 0.857 | 1.0593 | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 | 3.467 | 3.745 | 4.382 |
| 25 | 0.256 | 0.531 | 0.856 | 1.0584 | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 | 3.450 | 3.725 | 4.352 |
| 26 | 0.256 | 0.531 | 0.856 | 1.0575 | 1.315 | 1.706 | 2.056 | 2.479 | 2.779 | 3.435 | 3.707 | 4.324 |
| 27 | 0.256 | 0.531 | 0.855 | 1.0567 | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 | 3.421 | 3.690 | 4.299 |
| 28 | 0.256 | 0.530 | 0.855 | 1.0560 | 1.313 | 1.701 | 2.048 | 2.467 | 2.763 | 3.408 | 3.674 | 4.275 |
| 29 | 0.256 | 0.530 | 0.854 | 1.0553 | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 | 3.396 | 3.659 | 4.254 |
| 30 | 0.256 | 0.530 | 0.854 | 1.0547 | 1.310 | 1.697 | 2.042 | 2.457 | 2.750 | 3.385 | 3.646 | 4.234 |
| 40 | 0.255 | 0.529 | 0.851 | 1.0500 | 1.303 | 1.684 | 2.021 | 2.423 | 2.704 | 3.307 | 3.551 | 4.094 |
| 60 | 0.254 | 0.527 | 0.848 | 1.0455 | 1.296 | 1.671 | 2.000 | 2.390 | 2.660 | 3.232 | 3.460 | 3.962 |
| 120 | 0.254 | 0.526 | 0.845 | 1.0409 | 1.289 | 1.658 | 1.980 | 2.358 | 2.617 | 3.160 | 3.373 | 3.837 |
| $\infty$ | 0.253 | 0.524 | 0.842 | 1.0364 | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 | 3.090 | 3.291 | 3.719 |

| Name | Notation | pdf/pmf | Range | Mean $\mu$ | Variance $\sigma^2$ | |
|---|---|---|---|---|---|---|
| **Beta** | $\mathsf{Be}(\alpha,\beta)$ | $f(x)=\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}x^{\alpha-1}(1-x)^{\beta-1}$ | $x\in(0,1)$ | $\frac{\alpha}{\alpha+\beta}$ | $\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$ | |
| **Binomial** | $\mathsf{Bi}(n,p)$ | $f(x)=\binom{n}{x}p^x q^{(n-x)}$ | $x\in 0,\cdots,n$ | $n\,p$ | $n\,p\,q$ | $(q=1-p)$ |
| **Exponential** | $\mathsf{Ex}(\lambda)$ | $f(x)=\lambda\,e^{-\lambda x}$ | $x\in\mathbb{R}_+$ | $1/\lambda$ | $1/\lambda^2$ | |
| **Gamma** | $\mathsf{Ga}(\alpha,\lambda)$ | $f(x)=\frac{\lambda^\alpha}{\Gamma(\alpha)}x^{\alpha-1}\,e^{-\lambda x}$ | $x\in\mathbb{R}_+$ | $\alpha/\lambda$ | $\alpha/\lambda^2$ | |
| **Geometric** | $\mathsf{Ge}(p)$ | $f(x)=p\,q^x$ | $x\in\mathbb{Z}_+$ | $q/p$ | $q/p^2$ | $(q=1-p)$ |
| | | $f(y)=p\,q^{y-1}$ | $y\in\{1,...\}$ | $1/p$ | $q/p^2$ | $(y=x+1)$ |
| **HyperGeo.** | $\mathsf{HG}(n,A,B)$ | $f(x)=\frac{\binom{A}{x}\binom{B}{n-x}}{\binom{A+B}{n}}$ | $x\in 0,\cdots,n$ | $n\,P$ | $n\,P\,(1-P)\frac{N-n}{N-1}$ | $(P=\frac{A}{A+B})$ |
| **Logistic** | $\mathsf{Lo}(\mu,\beta)$ | $f(x)=\frac{e^{-(x-\mu)/\beta}}{\beta[1+e^{-(x-\mu)/\beta}]^2}$ | $x\in\mathbb{R}$ | $\mu$ | $\pi^2\beta^2/3$ | |
| **Log Normal** | $\mathsf{LN}(\mu,\sigma^2)$ | $f(x)=\frac{1}{x\sqrt{2\pi\sigma^2}}e^{-(\log x-\mu)^2/2\sigma^2}$ | $x\in\mathbb{R}_+$ | $e^{\mu+\sigma^2/2}$ | $e^{2\mu+\sigma^2}\left(e^{\sigma^2}-1\right)$ | |
| **Neg. Binom.** | $\mathsf{NB}(\alpha,p)$ | $f(x)=\binom{x+\alpha-1}{x}p^\alpha\,q^x$ | $x\in\mathbb{Z}_+$ | $\alpha q/p$ | $\alpha q/p^2$ | $(q=1-p)$ |
| | | $f(y)=\binom{y-1}{y-\alpha}p^\alpha\,q^{y-\alpha}$ | $y\in\{\alpha,...\}$ | $\alpha/p$ | $\alpha q/p^2$ | $(y=x+\alpha)$ |
| **Normal** | $\mathsf{No}(\mu,\sigma^2)$ | $f(x)=\frac{1}{\sqrt{2\pi\sigma^2}}e^{-(x-\mu)^2/2\sigma^2}$ | $x\in\mathbb{R}$ | $\mu$ | $\sigma^2$ | |
| **Pareto** | $\mathsf{Pa}(\alpha,\epsilon)$ | $f(x)=\alpha\,\epsilon^\alpha/x^{\alpha+1}$ | $x\in(\epsilon,\infty)$ | $\frac{\epsilon\,\alpha}{\alpha-1}$ | $\frac{\epsilon^2\alpha}{(\alpha-1)^2(\alpha-2)}$ | |
| **Poisson** | $\mathsf{Po}(\lambda)$ | $f(x)=\frac{\lambda^x}{x!}e^{-\lambda}$ | $x\in\mathbb{Z}_+$ | $\lambda$ | $\lambda$ | |
| **Snedecor** $F$ | $F(\nu_1,\nu_2)$ | $f(x)=\frac{\Gamma(\frac{\nu_1+\nu_2}{2})(\nu_1/\nu_2)^{\nu_1/2}}{\Gamma(\frac{\nu_1}{2})\Gamma(\frac{\nu_2}{2})}\times x^{\frac{\nu_1-2}{2}}\left[1+\frac{\nu_1}{\nu_2}x\right]^{-\frac{\nu_1+\nu_2}{2}}$ | $x\in\mathbb{R}_+$ | $\frac{\nu_2}{\nu_2-2}$ | $\left(\frac{\nu_2}{\nu_2-2}\right)^2\frac{2(\nu_1+\nu_2-2)}{\nu_1(\nu_2-4)}$ | |
| **Student** $t$ | $t_\nu$ | $f(x)=\frac{\Gamma(\frac{\nu+1}{2})}{\Gamma(\frac{\nu}{2})\sqrt{\pi\nu}}[1+x^2/\nu]^{-(\nu+1)/2}$ | $x\in\mathbb{R}$ | $0$ | $\nu/(\nu-2)$ | |
| **Uniform** | $\mathsf{Un}(a,b)$ | $f(x)=\frac{1}{b-a}$ | $x\in(a,b)$ | $\frac{a+b}{2}$ | $\frac{(b-a)^2}{12}$ | |
| **Weibull** | $\mathsf{We}(\alpha,\beta)$ | $f(x)=\alpha\beta\,x^{\alpha-1}\,e^{-\beta\,x^\alpha}$ | $x\in\mathbb{R}_+$ | $\frac{\Gamma(1+\alpha^{-1})}{\beta^{1/\alpha}}$ | $\frac{\Gamma(1+2/\alpha)-\Gamma^2(1+1/\alpha)}{\beta^{2/\alpha}}$ | |

Name: ____________

Mth 136 = Sta 114