Midterm Examination #1

Mth 135 = Sta 104

Tuesday, October 18, 2005
10:05 – 11:20 am
Version *a*

- This is a **closed book** exam— put your books on the floor.
- You may use a calculator and **one page** of your own notes.
- Do not share calculators or notes.
- **Show** your **work**. Neatness counts.
- Numerical answers: **four significant digits** or fractions **in lowest terms**.
- PDF and normal distribution tables are attached.

Cheating on exams is a breach of trust with classmates and faculty that will not be tolerated. After completing the exam please acknowledge that you have adhered to the Duke Community Standard:

- I will not lie, cheat, or steal in my academic endeavors,
  nor will I accept the actions of those who do.
- I will conduct myself responsibly and honorably in all my
  activities as a Duke student.

Signature: ____________________

Print Name: ____________________

1.	/20
2.	/20
3.	/20
4.	/20
5.	/20
Total:	/100

1. Mick chooses a number M uniformly from $\{1,2,3\}$, Keith chooses K uniformly from $\{1,2,3,4\}$, and Ron chooses R uniformly from $\{1,2,3,4,5\}$, independently— you might like to think of these as the number of dots showing on fair n-sided dice for $n=3,4,5$, respectively. Find the following probabilities; show your work.

a. (5) $\mathsf{P}[M+K+R=3]=$

b. (5) $\mathsf{P}[M=R]=$

c. (5) $\mathsf{P}[M>R]=$

d. (5) $\mathsf{P}[M>R \mid M\geq R]=$

2. Choose the best probability distribution for each random variable below from among the choices *Binomial*, *Exponential*, *Gamma*, *Geometric*, *Hypergeometric*, *Negative Binomial*, *Normal*, *Poisson*, or *Uniform* and, whatever the distribution, give its mean μ:

a) (4) The number of times Mick Jagger struts out to the edge of the stage successfully before the first time he falls, if falls are independent and have probability 0.01:
○ Bi ○ Ex ○ Ga ○ Ge ○ HG ○ NB ○ No ○ Po ○ Un
$\mu =$

b) (4) The number of Rolling Stones wearing boxer shorts, if each of the four musicians chooses "boxes or briefs" independently by tossing a fair coin:
○ Bi ○ Ex ○ Ga ○ Ge ○ HG ○ NB ○ No ○ Po ○ Un
$\mu =$

c) (4) The number of Duke students in the first row, if the first ten rows hold a total of 500 concert-goers including 40 Duke students, and if all rows hold the same number of people. What assumption did you make?
○ Bi ○ Ex ○ Ga ○ Ge ○ HG ○ NB ○ No ○ Po ○ Un
$\mu =$ Assume:

d) (4) The number of times Charlie Watts nods to the crowd during the two-hour concert, if he nods about once every five minutes and if the numbers of nods in different time periods are independent?
○ Bi ○ Ex ○ Ga ○ Ge ○ HG ○ NB ○ No ○ Po ○ Un
$\mu =$

e) (4) The total amount of beer spilled on the floor of Wallace Wade, if 10 000 beers were sold and if 0.01 ounces are spilled on average from each beer, with a standard deviation of 0.10 ounces?
○ Bi ○ Ex ○ Ga ○ Ge ○ HG ○ NB ○ No ○ Po ○ Un
$\mu =$

3. Keith Richards rolls a fair die repeatedly and counts the number K of non-aces before he first rolls an ace. Ron Woods tosses three fair coins and counts the number R of times they show different faces before the first time they all show the same face. Find:

a. (10) $\mathsf{P}[K > R] =$

b. (10) For every real number $x \in \mathbb{R}$, find the probability that the *minimum* of K and R is exactly equal to x:
$\mathsf{P}[\min(K, R) = x]$

4. Charlie Watts draws W uniformly from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Find:

a. (4) The probability that W^2 is odd:
$\mathsf{P}[W^2 \in \{1, 3, 5, ...\}] =$

b. (12) The following expectations and variance:[1]

$\mathsf{E}[(-1)^W] =$ ______________ $\mathsf{E}[2^W] =$ ______________

$\mathsf{E}[W] =$ ______________ $\mathsf{Var}[W] =$ ______________

$\mathsf{E}[4W-1] =$ ______________ $\mathsf{Var}[4W-1] =$ ______________

c. (4) If Charlie repeats this experiment 100 times, making independent draws $W_1, ..., W_{100}$, find the approximate probability that the sum $S_{100} = \sum W_i$ is at least 575:[1]
$\mathsf{P}[S_{100} \geq 575] \approx$

[1] Helpful hint: The discrete uniform distribution on $\{1, ..., n\}$ has mean $(n+1)/2$ and variance $(n^2-1)/12$.

5. A CD case contains six Rolling Stones CD's and four Pink Floyd CD's. All ten CD's are removed from the case, one at a time, in (uniform) random order. CD's are not replaced once they have been drawn.

a. (5) What is the probability that exactly two of the first three CD's are by the Stones?

$\mathsf{P}[\text{2 Stones in } 1^{\text{st}} \text{ 3 CD's}] =$

b. (5) If the first three CD's are all by the same band, what is the probability that the band is the Rolling Stones?

$\mathsf{P}[\text{Stones} \mid 1^{\text{st}} \text{ 3 Match}] =$

c. (10) What is the expected value of S, the number of Stones CD's drawn *before* any Floyd CD is drawn? (One way to do it: write $S = \sum I_i$, where the indicator I_i is **one** if the i^{th} Stones CD is drawn before any of the Floyd CD's, and **zero** otherwise. What is $\mathsf{E}[I_i]$?).

$\mathsf{E}[S] =$

Extra worksheet, if needed:

$$\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-z^2/2}\, dz:$$

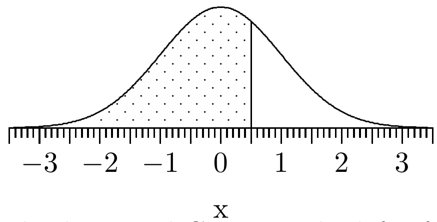

Table 5.1 Area $\Phi(x)$ under the Standard Normal Curve to the left of x.

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

$\Phi(0.6745) = 0.75$ $\Phi(1.6449) = 0.95$ $\Phi(2.3263) = 0.99$ $\Phi(3.0902) = 0.999$

$\Phi(1.2816) = 0.90$ $\Phi(1.9600) = 0.975$ $\Phi(2.5758) = 0.995$ $\Phi(3.2905) = 0.9995$

Name	Notation	pdf/pmf	Range	Mean μ	Variance σ^2	
Beta	$\mathsf{Be}(\alpha,\beta)$	$f(x)=\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}x^{\alpha-1}(1-x)^{\beta-1}$	$x\in(0,1)$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$	
Binomial	$\mathsf{Bi}(n,p)$	$f(x)=\binom{n}{x}p^x q^{(n-x)}$	$x\in\{0,\cdots,n\}$	$n\,p$	$n\,p\,q$	$(q=1-p)$
Exponential	$\mathsf{Ex}(\lambda)$	$f(x)=\lambda\,e^{-\lambda x}$	$x\in\mathbb{R}_+$	$1/\lambda$	$1/\lambda^2$	
Gamma	$\mathsf{Ga}(\alpha,\lambda)$	$f(x)=\frac{\lambda^\alpha}{\Gamma(\alpha)}x^{\alpha-1}\,e^{-\lambda x}$	$x\in\mathbb{R}_+$	α/λ	α/λ^2	
Geometric	$\mathsf{Ge}(p)$	$f(x)=p\,q^x$	$x\in\mathbb{Z}_+$	q/p	q/p^2	$(q=1-p)$
		$f(y)=p\,q^{y-1}$	$y\in\{1,\cdots\}$	$1/p$	q/p^2	$(y=x+1)$
HyperGeo.	$\mathsf{HG}(n,A,B)$	$f(x)=\frac{\binom{A}{x}\binom{B}{n-x}}{\binom{A+B}{n}}$	$x\in\{0,\cdots,n\}$	$n\,P$	$n\,P\,(1-P)\frac{N-n}{N-1}$	$(P=\frac{A}{A+B})$
Logistic	$\mathsf{Lo}(\mu,\beta)$	$f(x)=\frac{e^{-(x-\mu)/\beta}}{\beta[1+e^{-(x-\mu)/\beta}]^2}$	$x\in\mathbb{R}$	μ	$\pi^2\beta^2/3$	
Log Normal	$\mathsf{LN}(\mu,\sigma^2)$	$f(x)=\frac{1}{x\sqrt{2\pi\sigma^2}}e^{-(\log x-\mu)^2/2\sigma^2}$	$x\in\mathbb{R}_+$	$e^{\mu+\sigma^2/2}$	$e^{2\mu+\sigma^2}\left(1-e^{\sigma^2}\right)$	
Neg. Binom.	$\mathsf{NB}(\alpha,p)$	$f(x)=\binom{x+\alpha-1}{x}p^\alpha\,q^x$	$x\in\mathbb{Z}_+$	$\alpha q/p$	$\alpha q/p^2$	$(q=1-p)$
		$f(y)=\binom{y-1}{y-\alpha}p^\alpha\,q^{y-\alpha}$	$y\in\{\alpha,\cdots\}$	α/p	$\alpha q/p^2$	$(y=x+\alpha)$
Normal	$\mathsf{No}(\mu,\sigma^2)$	$f(x)=\frac{1}{\sqrt{2\pi\sigma^2}}e^{-(x-\mu)^2/2\sigma^2}$	$x\in\mathbb{R}$	μ	σ^2	
Pareto	$\mathsf{Pa}(\alpha,\beta)$	$f(x)=\beta\,\alpha^\beta/x^{\beta+1}$	$x\in(\alpha,\infty)$	$\frac{\alpha\,\beta}{\beta-1}$	$\frac{\alpha^2\beta}{(\beta-1)^2(\beta-2)}$	
Poisson	$\mathsf{Po}(\lambda)$	$f(x)=\frac{\lambda^x}{x!}e^{-\lambda}$	$x\in\mathbb{Z}_+$	λ	λ	
Student t	$t(\nu)$	$f(x)=\frac{\Gamma(\frac{\nu+1}{2})}{\Gamma(\frac{\nu}{2})\sqrt{\pi\nu}}[1+x^2/\nu]^{-(\nu+1)/2}$	$x\in\mathbb{R}$	0	$\nu/(\nu-2)$	
Unif. (Cont)	$\mathsf{Un}(a,b)$	$f(x)=\frac{1}{b-a}$	$x\in(a,b)$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	
Unif. (Disc)	$\mathsf{Un}\{a,\cdots,b\}$	$f(x)=\frac{1}{b-a+1}$	$x\in\{a,\cdots,b\}$	$\frac{a+b}{2}$	$\frac{(b-a)(b-a+2)}{12}$	
Weibull	$\mathsf{We}(\alpha,\beta,\gamma)$	$f(x)=\frac{\alpha(x-\gamma)^{\alpha-1}}{\beta^\alpha}e^{-[(x-\gamma)/\beta]^\alpha}$	$x\in(\gamma,\infty)$	$\gamma+\beta\Gamma(1+\alpha^{-1})$		

Name: ____________

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