

Midterm Examination # 2

Mth 135 = Sta 104

Monday, November 15, 2004
90 Minutes

This is a closed-book exam. You may use a single one-sided sheet of *your own* notes, if you wish. If you don't understand something in one of the questions, please ask *me*. You may use a calculator (only) if you wish.

You must **show** your **work** to receive credit. Please give all numerical answers as fractions **in lowest terms** (simplify!) or as decimals correct to **four places**.

Cheating on exams is a breach of trust with classmates and faculty, and will not be tolerated. After completing the exam please acknowledge that you have adhered to the Duke Community Standard:

- I will not lie, cheat, or steal in my academic endeavors, nor will I accept the actions of those who do.
- I will conduct myself responsibly and honorably in all my activities as a Duke student.

Signature: _____

Print Name: _____

1.	/20
2.	/20
3.	/20
4.	/20
5.	/20
Total:	/100

Name: _____

Mth 135 = Sta 104

1. The independent random variables X and Y have uniform distributions on the interval $[0, 1]$. Find:

a. (3) $P[X > 3Y] =$

b. (3) $E[X^3 + Y^3] =$

c. (5) $E[e^{3X}] =$

d. (3) $E[X \times Y] =$

e. (3) $E[e^{X-Y}] =$

f. (3) $E[\sqrt{X/Y}] =$

Hint: *No* 2-dimensional integration is needed, just a little plane geometry.

2. The random variables X and Y have a joint probability density function given by

$$f(x, y) = \begin{cases} 6(y - x) & 0 < x < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

Find:

a. (8) The marginal probability density functions:

$$f_1(x) = \qquad \qquad \qquad f_2(y) =$$

b. (4) $P[Y < 0.50]$

c. (3) $P[X < Y]$

d. (5) For some positive integers j , k , and n , X and Y have the same distribution as the j 'th and k 'th smallest $U_{(j)}$ and $U_{(k)}$ of n independent uniformly distributed random variables U_1, \dots, U_n . What are the values of j , k , and n ?

$$j = \underline{\hspace{2cm}} \quad k = \underline{\hspace{2cm}} \quad n = \underline{\hspace{2cm}}$$

Name: _____

Mth 135 = Sta 104

3. The random variables X and Y have independent normal distributions, with means and variances given by:

$$E[X] = 1 \quad \text{Var}[X] = 9 \quad E[Y] = 2 \quad \text{Var}[Y] = 16.$$

Without doing any integration, find each of the following:

a. (5) $E[X^2 + Y^2] =$ _____

b. (5) $P[X < 9 + Y] =$ _____

c. (5) $\text{Var}[2X - Y] =$ _____

d. (5) $E[X \times Y] =$ _____

4. The random variable X has an exponential distribution with mean $1/\ln 4$, so it has probability density function and cumulative distribution function

$$f(x) = \begin{cases} 0 & \text{for } x \leq 0 \\ \lambda e^{-\lambda x} & \text{for } x > 0 \end{cases} \quad F(x) = \begin{cases} 0 & \text{for } x \leq 0 \\ 1 - e^{-\lambda x} & \text{for } x > 0 \end{cases}$$

with $\lambda = \ln 4$, so $P[X > x] = 4^{-x}$ for all $x \in (0, \infty)$. The random variable $Y \equiv \lfloor X \rfloor$ is X , rounded *down* to the next lower integer... so if X is 3, π , or 3.9999999 then $Y = 3$, but if $X = 4.00000001$ then $Y = 4$.

- a. (5) Does Y have a *discrete* distribution or a *continuous* one?
Circle one: Disc Cont and find the p.m.f. or p.d.f.:

$$f(y) =$$

- b. (5) The distribution of Y is one of the ones we've studied— what is its name, the value of any parameter(s), and its mean? (see pg. 8)

- c. (5) Find the indicated probability:

$$P[Y \geq 2] =$$

- d. (5) Find the conditional probability:

$$P[X \leq 2.5 | Y \geq 2] =$$

5. The lifetimes (in months) before quixleblats fail are random variables with survival function $\Pr[T > t] = e^{-\sqrt{t}}$, for $t > 0$.

- a. (10) Pat has two quixleblats— a brand new one, and a one-month old one. Which has a better chance of lasting for one more month? Why? Show your work.

Check one: New Old Doesn't Matter

Reasoning:

- b. (5) Find the probability density function for quixleblat life at *all* $t \in \mathbb{R}$:

$$f_T(t) = \underline{\hspace{2cm}}$$

- c. (5) Find the probability density function for $X = \sqrt{T}$ at *all* $x \in \mathbb{R}$:

$$f_X(x) = \underline{\hspace{2cm}}$$

Name: _____

Mth 135 = Sta 104

Extra worksheet, if needed:

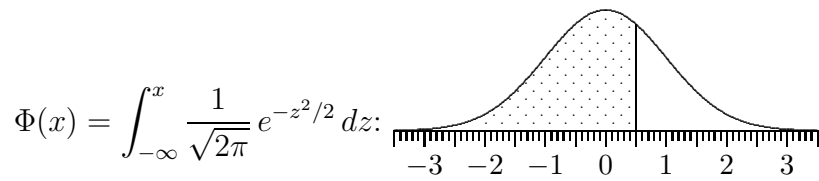


Table 5.1 Area $\Phi(x)$ under the Standard Normal Curve to the left of x .

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

$\Phi(0.6745) = 0.75$ $\Phi(1.6449) = 0.95$ $\Phi(2.3263) = 0.99$ $\Phi(3.0902) = 0.999$
 $\Phi(1.2816) = 0.90$ $\Phi(1.9600) = 0.975$ $\Phi(2.5758) = 0.995$ $\Phi(3.2905) = 0.9995$

Name	Notation	pdf/pmf	Range	Mean μ	Variance σ^2
Beta	$\text{Be}(\alpha, \beta)$	$f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1} (1-x)^{\beta-1}$	$x \in (0, 1)$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$
Binomial	$\text{Bi}(n, p)$	$f(x) = \binom{n}{x} p^x q^{(n-x)}$	$x \in 0, \dots, n$	np	$npq \quad (q = 1 - p)$
Exponential	$\text{Ex}(\lambda)$	$f(x) = \lambda e^{-\lambda x}$	$x \in \mathbb{R}_+$	$1/\lambda$	$1/\lambda^2$
Gamma	$\text{Ga}(\alpha, \lambda)$	$f(x) = \frac{\lambda^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x}$	$x \in \mathbb{R}_+$	α/λ	α/λ^2
Geometric	$\text{Ge}(p)$	$f(x) = p q^x$ $f(y) = p q^{y-1}$	$x \in \mathbb{Z}_+$ $y \in \{1, \dots\}$	q/p $1/p$	$q/p^2 \quad (q = 1 - p)$ $q/p^2 \quad (y = x + 1)$
HyperGeo.	$\text{HG}(n, A, B)$	$f(x) = \frac{\binom{A}{x} \binom{B}{n-x}}{\binom{A+B}{n}}$	$x \in 0, \dots, n$	nP	$nP(1-P) \frac{N-n}{N-1} \quad (P = \frac{A}{A+B})$
Logistic	$\text{Lo}(\mu, \beta)$	$f(x) = \frac{e^{-(x-\mu)/\beta}}{\beta[1+e^{-(x-\mu)/\beta}]^2}$	$x \in \mathbb{R}$	μ	$\pi^2 \beta^2 / 3$
Log Normal	$\text{LN}(\mu, \sigma^2)$	$f(x) = \frac{1}{x\sqrt{2\pi\sigma^2}} e^{-(\log x - \mu)^2 / 2\sigma^2}$	$x \in \mathbb{R}_+$	$e^{\mu + \sigma^2/2}$	$e^{2\mu + \sigma^2} (e^{\sigma^2} - 1)$
Neg. Binom.	$\text{NB}(\alpha, p)$	$f(x) = \binom{x+\alpha-1}{x} p^\alpha q^x$ $f(y) = \binom{y-1}{y-\alpha} p^\alpha q^{y-\alpha}$	$x \in \mathbb{Z}_+$ $y \in \{\alpha, \dots\}$	$\alpha q/p$ α/p	$\alpha q/p^2 \quad (q = 1 - p)$ $\alpha q/p^2 \quad (y = x + \alpha)$
Normal	$\text{No}(\mu, \sigma^2)$	$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2 / 2\sigma^2}$	$x \in \mathbb{R}$	μ	σ^2
Pareto	$\text{Pa}(\alpha, \epsilon)$	$f(x) = \alpha \epsilon^\alpha / x^{\alpha+1}$	$x \in (\epsilon, \infty)$	$\frac{\epsilon \alpha}{\alpha-1}$	$\frac{\epsilon^2 \alpha}{(\alpha-1)^2 (\alpha-2)}$
Poisson	$\text{Po}(\lambda)$	$f(x) = \frac{\lambda^x}{x!} e^{-\lambda}$	$x \in \mathbb{Z}_+$	λ	λ
Snedecor F	$F(\nu_1, \nu_2)$	$f(x) = \frac{\Gamma(\frac{\nu_1+\nu_2}{2}) (\nu_1/\nu_2)^{\nu_1/2}}{\Gamma(\frac{\nu_1}{2}) \Gamma(\frac{\nu_2}{2})} \times$ $x^{\frac{\nu_1-2}{2}} \left[1 + \frac{\nu_1}{\nu_2} x \right]^{-\frac{\nu_1+\nu_2}{2}}$	$x \in \mathbb{R}_+$	$\frac{\nu_2}{\nu_2-2}$	$\left(\frac{\nu_2}{\nu_2-2} \right)^2 \frac{2(\nu_1+\nu_2-2)}{\nu_1(\nu_2-4)}$
Student t	t_ν	$f(x) = \frac{\Gamma(\frac{\nu+1}{2})}{\Gamma(\frac{\nu}{2}) \sqrt{\pi\nu}} [1 + x^2/\nu]^{-(\nu+1)/2}$	$x \in \mathbb{R}$	0	$\nu/(\nu-2)$
Uniform	$\text{Un}(a, b)$	$f(x) = \frac{1}{b-a}$	$x \in (a, b)$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
Weibull	$\text{We}(\alpha, \beta)$	$f(x) = \alpha \beta x^{\alpha-1} e^{-\beta x^\alpha}$	$x \in \mathbb{R}_+$	$\frac{\Gamma(1+\alpha^{-1})}{\beta^{1/\alpha}}$	$\frac{\Gamma(1+2/\alpha) - \Gamma^2(1+1/\alpha)}{\beta^{2/\alpha}}$

Name: _____

Mth 135 = Sta 104

Fall 2004
November 5, 2004