

- [Aug 30, Sep 1](#) : Probability
Reading: chap 2
HW: 26, 30, 33, 48, 59, 61, 69, 78, 80.

26. A certain system can experience three different types of defects. Let A_i ($i=1, 2, 3$) denote the event that the system has a defect of type i . Suppose that

$$P(A_1)=0.12 \quad P(A_2)=0.07 \quad P(A_3)=0.05$$

$$P(A_1 \cup A_2)=0.13 \quad P(A_1 \cup A_3)=0.14$$

$$P(A_2 \cup A_3)=0.10 \quad P(A_1 \cap A_2 \cap A_3)=0.01$$

- What is the probability that the system does not have a type 1 defect?
- What is the probability that the system has both type 1 and type 2 defects?
- What is the probability that the system has both type 1 and type 2 defects but not a type 3 defect?
- What is the probability that the system has at most two of these defects?

30. A friend of mine is giving a dinner party. His current wine supply includes 8 bottles of zinfandel, 10 of merlot, and 12 of cabernet (he only drinks red wine), all from different wineries.

- If he wants to serve 3 bottles of zinfandel and serving order is important, how many ways are there to do this?
- If 6 bottles of wine are to be randomly selected from the 30 for serving, how many ways are there to do this?
- If 6 bottles are randomly selected, how many ways are there to obtain two bottles of each variety?
- If 6 bottles are randomly selected, what is the probability that this results in two bottles of each variety being chosen?
- If 6 bottles are randomly selected, what is the probability that all of them are the same variety.

33. Shortly after being put into service, some buses manufactured by a certain company have developed cracks on the underside of the main frame. Suppose a particular city has 25 of these buses, and cracks have actually appeared in 8 of them.

- How many ways are there to select a sample of 5 buses from the 25 for a thorough inspection?
- In how many ways can a sample of 5 buses contain exactly 4 with visible cracks?
- If a sample of 5 buses is chosen at random, what is the probability that exactly 4 of the 5 will have visible cracks?
- If buses are selected as in part (c.), what is the probability that at least 4 of those selected will have visible cracks?

48. Reconsider the system defect situation described in Exercise 26 (section 2.2).

- Given that the system has a type 1 defect, what is the probability that it has a type 2 defect?
- Given that the system has a type 1 defect, what is the probability that it has all three types of defects?

- c. Given that the system has at least one type of defect, what is the probability that it has exactly one type of defect?
- d. Given that the system has both of the first two types of defect, what is the probability that it does not have the third type of defect?

59. At a certain gas station, 40% of the customers use regular unleaded gas (A_1), 35% use extra unleaded gas (A_2), and 25% use premium unleaded gas (A_3). Of those customers using regular gas, only 30% fill their tanks (event B). Of those customers using extra gas, 60% fill their tanks, whereas of those using premium, 50% fill their tanks.

- a. What is the probability that the next customer will request extra unleaded gas and fill the tank ($A_2 \cap B$)?
- b. What is the probability that the next customer fills the tank?
- c. If the next customer fills the tank, what is the probability that regular gas is requested? Extra gas? Premium gas?

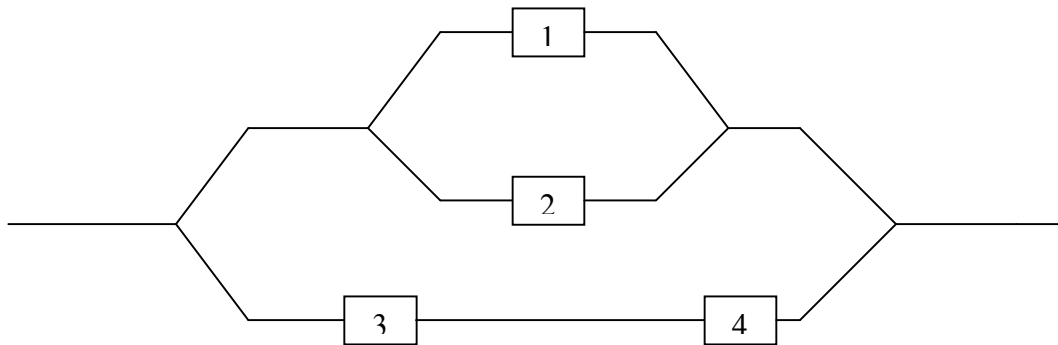
61. Components of a certain type are shipped to a supplier in batches of ten. Suppose that 50% of all such batches contain no defective components, 30% contain one defective component, and 20% contain two defective components. Two components from a batch are randomly selected and tested. What are the probabilities associated with 0, 1, and 2 defective components being in the batch under each of the following conditions?

- a. Neither tested component is defective.
- b. One of the two tested components is defective. (Hint: Draw a tree diagram with three first-generation branches for the three different types of batches.)

69. An oil exploration company currently has two active projects, one in Asia and the other in Europe. Let A be the event that the Asian project is successful and B be the event that the European project is successful. Suppose that A and B are independent events with $P(A)=0.4$ and $P(B)=0.7$.

- a. If the Asian project is not successful, what is the probability that the European project is also not successful? Explain your reasoning.
- b. What is the probability that at least one of the two projects will be successful?
- c. Given that at least one of the two projects is successful, what is the probability that only the Asian project is successful?

78. Consider the system of components connected as in the accompanying picture. Components 1 and 2 are connected in parallel, so that subsystem works iff either 1 or 2 works; since 3 and 4 are connected in series, that subsystem works iff both 3 and 4 work. If components work independently of one another and $P(\text{component works})=0.9$, calculate $P(\text{system works})$.



80. Consider independently rolling two fair dice, one red and the other green. Let A be the event that the red die shows 3 dots, B be the event that the green die shows 4 dots, and C be the event that the total number of dots showing on the two dice is 7. Are these events pairwise independent (i.e., are A and B independent events, are A and C independent, and are B and C independent)? Are the three events mutually independent?